Hierarchy of Density Dependent Scalar Fields in Gravitational Models

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Abstract: This paper discusses the connection between dark matter and the gravitational constant through a hierarchy of density dependent scalar fields from the far Universe down to the laboratory scale. Such hierarchy implies that the gravitational constant could vary under certain conditions where the field density changes from the local field density. This would allow for force variations in classical gravitational models that could lead to new force methods for terrestrial applications as well as understanding new astrophysical observations. Using Chameleon Cosmology, a dark matter model that is fundamentally dependent on the background density, a new hierarchy interpretation of the gravitational constant G is produced. This is accomplished by equating the far field Chameleon particle mass to the dark matter particle mass derived from observed astronomical data. It is shown that the gravitational constant G can be given in terms of the dark matter particle mass and the hierarchy of the background fields - represented by a hierarchy of coupling constants. This relationship provides a starting point for using new galactic structures as guidance for modifying existing gravitational models, as quantum gravity, or defining new models, which could be verifiable in experiments on the laboratory scale.

1. INTRODUCTION

The overwhelming preponderance of matter in the universe is believed to be dark. Therefore, one should expect dark matter to play a fundamental role in the attraction of masses even on the laboratory scale. This suggest that there is a hierarchy of scalar fields in the universe with properties varying with background conditions that should affect force measurement compared to classical gravitational models. In this paper, scalar field concepts based on astronomical objects are applied to the laboratory scale by connecting the dark matter particle mass derived from observed astronomical data to the gravitational constant G. This is accomplished using the Chameleon Cosmology [1, 2], which address a scalar field coupled directly to matter with respect to the surrounding density field. Whereby, galactic structures attributed to dark matter can be compared with the universe background density field to provide guidance for laboratory experiments in our local background density field.

Further, hierarchy of scalar fields in the universe may also answer the question: *Can gravity remain fundamentally classical while interacting with quantum fields.* After all, as far as all our experiments show: gravity is classical and matter is quantum. Section 6, of this paper discusses the hierarchy of scalar fields in the universe in relation to quantum gravity.

2.0. THE CHAMELEON THEORY

Chameleon Cosmology [1, 2] assumes that our Universe is composed of a nearly massless scalar field. From a theoretical standpoint, massless scalar fields are abundant in string and supergravity theories. Indeed, generic compactifications of string theory result in a plethora of massless scalars in the low-energy - four dimensional - effective theory. However, these massless fields generally couple directly to matter with gravitational strength, and therefore lead to unacceptably large violations of the equivalence principle. That is, the Universe scalar field is essentially massless on solar system scales, and therefore subject to tight constraints from tests of the equivalence principle (For a review of experimental tests of the equivalence principle and general relativity, see [3, 4]).

In Chameleon Cosmology, scalar fields that have cosmological effects, such as quintessence, do not result in large violations of the equivalence principle in laboratories on earth because of its very dense environment. Thus, the main constraint on this model is that the mass of the field be sufficiently large on earth to evade equivalence principle and fifth force constraints (For a review of fifth force searches, see [5]). This is because the scalar field acquires a mass whose magnitude depends on the local density field. That is, in a region of high density, such as on earth, the mass of the field is large, and thus the resulting violations of the equivalence principle are exponentially suppressed. In the solar system, where the density is much lower, the fields are essentially free.

While the idea of a density-dependent mass term is not new [6 - 11], the novelty of Chameleon Cosmology lies in the fact that the scalar fields can couple directly to baryons with gravitational strength; suggesting a strong connection between Chameleon Cosmology and the gravitational constant.

2.1 Coupling Constants

A coupling constant is a number that determines the strength of an interaction. Usually the Lagrangian or the Hamiltonian of a system can be separated into a kinetic part and an interaction part. The coupling constant determines the strength of the interaction part with respect to the kinetic part, or between two sectors of the interaction part. For example, the electric charge (+ or –) of a particle is a coupling constant.

Coupling constants play an important role in dynamics. For example, one often sets up hierarchies of approximation based on the importance of various coupling constants. In the motion of a large lump of magnetized iron, the magnetic forces are more important than the gravitational forces because of the relative magnitudes of the coupling constants. However, in classical mechanics one usually makes these decisions directly by comparing forces.

In Chameleon Cosmology, the generation of a density-dependent mass for a given field ϕ results from the interplay of two source terms in its equation of motion, which leads to a Lagrangian dominated by an effective potential $V_{eff}(\phi) = V(\phi) + \rho e^{\beta\phi/M_{Pl}}$ containing these coupling terms [1, 2]; noted by β. These dimensionless coupling constants β need not be small but > 0 with values of order unity allowed, whereby in this paper $0 < \beta \leq 1$.

The first term arises from self interactions, described by a monotonically decreasing potential $V(\phi) = M^{4+n}/\phi^n$, which is of the runaway form. [The factor M is defined in equation (2)] In particular, the fact that the potential need not have a minimum; rather, it must be monotonic (*i.e.*, never increasing as the values of the independent variable or subscript terms increase accordingly). Noting from equations (5) and (22) that the background field ϕ is dependent on the coupling constants β.

The second term arises from the conformal coupling to density fields, of the form $\rho e^{\beta\phi/M_{Pl}}$, where the reduced Planck mass $M_{PL} = \sqrt{\hbar c / 8\pi G} = 4.34 \times 10^{-9}\, kg$, $G \sim 6.6728x10^{-11}\, m^3 / kg{\cdot}s^2$ is the gravitational constant, $\hbar = 1.0546x10^{-34}\, m^2{\cdot}kg / s$ is the reduced Planck constant, $c = 2.9979x10^8\, m / s$ is the speed of light. However unlike Chameleon Cosmology [1, 2], which basically ignores the effects of the coupling constants, in this analysis several coupling constants β are defined that are allowed to change with the hierarchy of the background fields. These coupling constants are shown to be a multiplier of the local density field and associated to the far-field though a dark matter hierarchy coupling constant β' that must be added as a multiplier of the local density ρ in the second term of the effective potential $V_{eff}(\phi)$ (see section: **4. HIERSRCHIES**). In effect, β' normalizes the conformal coupling to dark matter in the local background field to that of the far-field (see section: **2.4 The Far Field**). As is shown if such were not the case, the value of the gravitational constant would not be constant across the Universe (see section: **4.1 Gravitational Constant Hierarchy**).

2.3 The Density Field

Chameleon Cosmology [1, 2] implies that the Universe is composed of density dependent scalar fields with defined spherical densities down to the Universe's critical density $\rho_c \approx 1.11778x10^{-26}\, kg / m^3$ in the far field (see section: **2.4 The Far Field**). That is, given a region or field ϕ_m of spherical radius R, the density of the field is dependent on the sum of the total mass in the field with the center of the density field equivalent to the center of the distribution of the gravitational mass in the field. Then for an object placed between two fields of different densities, the field force on the object is a function of the change in the gradient $\nabla\phi_0$ between the two fields. As an addition to the classical gravitational model, the field force under normal conditions is much-much smaller than the classical gravitational force. Otherwise, it would be in contradiction to measured gravitational forces, the exception being Universe expansion.

Under this model, the acceleration of the Universe is caused by variations in density fields. That is, the field force in the far field where gravitational forces F_g are minimized is toward the smaller density field, *i.e.*, away from the bulk "gravitational" mass of the Universe. Such that in the far-field (noted by the subscript "c"), the far-field force $F_{\phi_c} > F_{g_c}$. However, near an object (noted by the subscript "m") this is reversed as the local field force $F_{\phi_m} \ll F_{g_m}$. This reversal comes from the conformal coupling of the local density field to the local density fields. Without such conformal coupling to density fields, the Universe would have accelerated at a near instantaneous rate from conception, *i.e.*, big bang.

In classical gravitational models areas of space where $F_{\phi_c} > F_{g_c}$ are flat and areas of space where $F_{\phi_m} \ll F_{g_m}$ are gravity wells.

2.4 The Far Field

In Chameleon Cosmology papers [1, 2], the coupling constants β are set to 1, *i.e.*, unity. This is assumed done as the analysis places objects in the near uniform density field of the earth, whereby objects couple directly with it. In the Universe field ϕ_c far from other objects, the dark matter particle masses exist in a similar state. Therefore the obvious choice for β_c in the far field ϕ_c would be 1. However, the earth's local field is motionless with respect to object placed in it. In the far field this is not the case as the far-field is accelerating, *i.e.*, Universe expansion.

The author has suggested that when objects or the ambient field's density are in motion, the motion coupling reduces $\beta < 1$ [12 - 14]. Since the universe is expanding, it is reasonable to assume that the far field density is in motion. Therefore, the far field ϕ_c coupling constant β_c is expected to be in the interval $0 < \beta_c \leq 1$.

The universe field ϕ_c far from any object is then chosen as the minimum density field having a (critical) density ρ_c and governed by an effective potential,

$$V_{eff}(\phi_c) = \frac{M_E^6}{\phi_c^2}\left(\frac{\hbar^3}{c}\right) + \rho_c c^2 exp^{\beta_c \phi_c M_{PL}}, \tag{1}$$

where the inverse length factor

$$M_E = \left(\frac{\Lambda}{8\pi l_{PL}^2}\right)^{\frac{1}{4}} \sim 10^4 m^{-1} \approx 2M\left(\frac{c}{\hbar}\right), \tag{2}$$

is used to remove the large scale energy scale interaction implied by M. That is, the inverse distance factor M_E implies an effective range of dark matter interactions in the far field and not the range of chameleon-mediated interaction, which implies a range on chameleon-mediated interactions that are very large (0.1 to 10^3 pc) in the far field [2]. The factor of 2 in equation (2) is needed to set the energy scale factor $M \sim 1.76x10^{-39} kg\ (\sim 10^{-3} eV / c^2)$, which is the upper bound associated with dark energy [15]. It is noted that from the above, the factor M implies that chameleon-mediated interactions in Chameleon Cosmology is actually dark energy.

In steady state, the far field ϕ_c attains a value which minimizes the energy density. The minimum is calculated by setting the derivative $dV/d\phi = 0$, as

$$\frac{dV_{eff}(\phi_c)}{d\phi_c} = -3\frac{M_E^6}{\phi_c^3}\left(\frac{\hbar^3}{c}\right) + \left(\frac{\beta_c \rho_c c^2}{M_{PL}}\right)\exp^{\beta_c \phi_c M_{PL}} = 0. \tag{3}$$

Now noting that the exponential

$$exp^{\beta_c \phi_c M_{PL}} = 1 + \beta_c \left(\frac{\phi_c}{M_{PL}} \right) + \frac{1}{2} \left(\beta_c \left(\frac{\phi_c}{M_{PL}} \right) \right)^2 \sim 1 \tag{4}$$

and assuming (at this point) that in the far field $\phi_c \ll M_{PL}$ and $0 < \beta \le 1$, equation (3) yields the far field

$$\phi_c = M_E^2 \left(\frac{3}{\beta_c} \frac{M_{PL}}{\rho_c} \right)^{1/3} \left(\frac{\hbar}{c} \right). \tag{5}$$

2.5 Far Field Chameleon Particle Mass

The Chameleon particle matter mass in the far field is given from

$$m_c^2 = \left(\frac{\hbar^3}{c^5} \right) \frac{dV_{eff}^2 (\phi_c)}{d\phi_c^2}, \tag{6}$$

which from equation (3) gives

$$m_c^2 = 12 \frac{M_E^6}{\phi_c^4} \left(\frac{\hbar}{c} \right)^6 + \frac{\beta_c^2 \rho_c}{M_{PL}^2} \left(\frac{\hbar}{c} \right)^3, \tag{7}$$

whereas before $e^{\beta_c \phi_c / M_{PL}} \sim 1$.

For $\phi_c \ll M_{PL}$,

$$12 \frac{M_E^6}{\phi_c^4} \left(\frac{\hbar}{c} \right)^6 \gg \frac{\beta_c^2 \rho_c}{M_{PL}^2} \left(\frac{\hbar}{c} \right)^3 \tag{8}$$

to give from equation (7)

$$\phi_c^4 \simeq 12 \frac{M_E^6}{m_c^2} \left(\frac{\hbar}{c} \right)^6, \tag{9}$$

which when combined with equation (5) yields

$$12 \frac{M_E^6}{m_c^2} \left(\frac{\hbar}{c} \right)^6 \simeq M_E^8 \left(\frac{3}{\beta_c} \frac{M_{PL}}{\rho_c} \right)^{4/3} \left(\frac{\hbar}{c} \right)^4 \tag{10}$$

or the far field Chameleon particle mass

$$m_c \simeq \frac{\sqrt{12}}{M_E} \left(\frac{\beta_c}{3} \frac{\rho_c}{M_{PL}} \right)^{2/3} \left(\frac{\hbar}{c} \right). \tag{11}$$

2.6 Estimating the Far Field Mass Coupling Constant β_c

The “Chameleon” is a postulated scalar particle with a non-linear self-interaction which gives the particle an effective mass that depends on its environment: the presence of other fields. It would have a small mass in much of intergalactic space, but a large mass in terrestrial experiments, making it difficult to detect. The chameleon is a possible candidate for dark energy and dark matter, and may contribute to cosmic inflation.

Given the above definition , in far space the Chameleon particle mass m_c should be expected to be equivalent to the dark matter particle mass. Whereby, the value of the dark matter particle mass formulated from natural observations of scalar fields in intergalactic space should be reasonably equivalent to the far field Chameleon particle mass m_c .

An example of a scalar field based on observational data of M31 (Andromeda galaxy) was developed in the context of Cold Dark Matter (CDM) by Silverman and Mallett [16], whom presented the case for coherent degenerate dark matter specific to a galactic superfluid where the equilibrium $dE/d\xi = 0$ between quantum pressure and gravitational attraction leads to a minimum size (the galactic coherence length) determined by the boson mass and condensate mass irrespective of the radial variation in density. Whereby, if the dark matter in M31 is due primarily to scalar bosons with a coherence length of the order of the size of the M31 luminous core, the dark matter particle mass $\sim 1.8340x10^{-23}\ eV / c^2 \approx 3.27002x10^{-59}\ kg$.

Letting the far field Chameleon particle mass m_c be equivalent $3.27002x10^{-59}\ kg$ and inputting it into equation (11), yields a value for the far field coupling constant $\beta_c \approx 0.1653$, which is within the assumption that $0 < \beta_c \leq 1$ for an accelerating far field.

Further, using these values gives the far field $\phi_c \simeq 6.930x10^{-29} kg$, using equation (5), which validates the assumptions that in the far field $\phi_c \ll M_{PL}$.

3. PLANCK, GRAVITATIONAL CONSTANT AND THE DARK MATTER PARTICLE MASS

To better understand the interplay of dark matter from a cosmological prospective, it is noted that the critical density is given in terms of the Hubble constant $H_0 \approx 2.5x10^{-18} s^{-1}$ as

$$\rho_c = \frac{3H_0^2}{8\pi G}. \tag{12}$$

Now noting $8\pi G = \hbar c / M_{PL}^2$ to remove the gravitational constant, yields the dark matter particle mass using equation (11) as

$$m_c = \left(\frac{\sqrt{12}}{M_E}\right)\left(\beta_c^2 H_0^4 M_{Pl}^2 \left(\frac{\hbar}{c^5}\right)\right)^{1/3} \sim 3.27002x10^{-59}\ kg, \tag{13}$$

for $\beta_c \approx 0.1653$, to yield the reduced Planck constant in terms of the far field Chameleon particle mass m_c as

$$\hbar = \frac{m_c^3}{\beta_c^2} \cdot R\, c^5, \tag{14}$$

where for simplicity

$$R = \left(\frac{1}{12\sqrt{12}} \right) \left(\frac{{M_E}^3}{{H_0}^4 {M_{Pl}}^2} \right) = 3.40x10^{97} s^4 / m^3 kg^2. \quad (15)$$

To recover the gravitational constant and remove the reduced Planck constant, it is noted that the Planck time

$$T_{PL} = \sqrt{\frac{\hbar G}{c^5}} \sim 5.4x10^{-44} \text{s}, \quad (16)$$

which when combined with equation (14) yields the gravitational constant as

$$G = \frac{\beta_c^2}{m_c^3} \cdot \frac{T_{PL}^2}{R}. \quad (17)$$

4. HIERARCHIES

Given equation (17), under classical gravitational models the ratio of the cubed dark matter particle mass m_c^3 to the squared coupling constant β_c^2 must be constant in all fields ϕ_m for the Gravitational constants G to remain – well constant. This implies a change to equation (11) as,

$$m_c^3 = \left(\frac{4}{3} \frac{\sqrt{12}}{M_E^3 \, M_{PL}^2} \frac{\hbar^3}{c^3} \right) \beta_c' \beta_c^2 \rho_c^2, \quad (18)$$

where β' is defined as the dark matter hierarchy coupling constant, which allows the dark matter particulate mass m_c to be the same in all fields, *i.e.*,

$$m_c^3 = \left(\frac{4}{3} \frac{\sqrt{12}}{M_E^3 \, M_{PL}^2} \frac{\hbar^3}{c^3} \right) \beta_m' \beta_m^2 \rho_m^2, \quad (19)$$

where from equations (18) and (19) (with $\beta_c' = 1$ as required for $m_c \approx 3.27002x10^{-59} \, kg$), it can be shown that the dark matter hierarchy coupling constant

$$\beta'_m = \frac{\beta_c^2}{\beta_m^2} \frac{\rho_c^2}{\rho_m^2} = \left(\frac{\beta_c}{\beta_m} \frac{\rho_c}{\rho_m} \right)^2. \quad (20)$$

Symmetry then implies that the hierarchial form of the minimum potential, equation (1), in any density dependent field ϕ_m be given as

$$V_{eff}\left(\phi_m\right) = \frac{M_E^6}{\phi_m^2} \left(\frac{\hbar^3}{c} \right) + \beta_m' \rho_m exp^{\beta_m \phi_m / M_{PL}}, \quad (21)$$

where it can be shown that the hierarchy of density dependent scalar fields in the universe can be given by

$$\phi_m = \phi_c \left(\frac{\beta_m}{\beta_c} \frac{\rho_m}{\rho_c} \right)^{1/3} = \frac{\phi_c}{\left(\sqrt{\beta_m'}\right)^{1/3}}. \quad (22)$$

4.1 Gravitational Constant Hierarchy

Since the Chameleon particle mass has *a small mass in much of intergalactic space*, but a *large mass in terrestrial experiments*, it seems reasonable to defined the Chameleon particle mass m_C for any field as

$$m_C = \left(\frac{m_c^3}{\beta'_m}\right)^{1/3} \geq m_c\,. \tag{23}$$

The hierarchy forms of the gravitational constant in each density field in the universe can now be given by combining equations (17), (20) and (23) to yield

$$G = \left(\frac{\beta'_m}{m_c^3}\right)\left(\frac{\beta_m^2 \rho_m^2}{\rho_c^2}\right)\frac{T_{PL}^2}{R} = \frac{1}{m_C^3}\left(\frac{\beta_c^2}{\beta'_m}\right)\frac{T_{PL}^2}{R} \approx \frac{1}{\beta'_m} \times 6.6799 \times 10^{-11}\ \ m^3/kg\ s^2\,. \tag{24}$$

Given that the Chameleon particle mass m_C and corresponding dark matter hierarchy coupling constant β'_m are different in each density field in the Universe and that as one moves through the Universe the density field changes (*i.e.*, overlap) as the surrounding density field changes due to the varying objects in the local density field, it is easy to assume that under certain condition in the laboratory, some non-terrestrial (*i.e.*, earth like) density fields could be produced. For example, it is known that parts of the Universe behave like a superconductor [*e.g.*, 17]. Since superconductors do exist in the local environment under extreme cold certain conditions, then one should assume that the Chameleon particle mass $m_C \to m_c$ in the Landau scalar field about a superconductor.

4.1.1 Gravitational Coupling Constant

Equation (23) implies a minimum coupling to local matter. That is, in the far field, where $\beta'_m = 1$, coupling to local matter

$$\leq m_C \approx m_c \sim 3.270x10^{-59}\,kg \tag{25}$$

becomes insignificant and implies that coupling to dark matter is strong. It further implies that quantum effects and other small scale effects may dominate the interactions with normal matter in the far field.

Near the earth $\beta_m \sim \beta_M \sim 1$, whereby from equation (20) with $\rho_m = 5520\,kg\,/\,m^3$, the earth's dark matter coupling constant $\beta'_M \sim 10^{-61}$ giving a large value for the Chameleon particle mass $m_C \approx 6.782x10^{-39}\,kg$ in terrestrial (earth) environments. This implies a very weak coupling to the dark matter particulate mass near the earth. That is, near the earth, coupling to local matter

$$\leq m_C \sim 6.782x10^{-39}\,kg \tag{26}$$

becomes insignificant, which is the order of quantum energies, such that quantum and other small scale effects are suppressed in the earth’s local density field.

Noting that equations (25) and (26) supports the definition at the beginning of sect 2.6.

It is noted that the value of the Chameleon particle mass m_C for any field is near equivalent to the gravitational coupling constant $\alpha_g = \left(m_i/m_p\right)^2$ for a specific mass in those fields, where m_i is an arbitrary mass and $m_p = 2.18\times10^{-8}\ kg$ is the Planck mass. However, the importance of such masses is left for further thought.

For example, by defining an arbitrary variable $\kappa \approx 1\,kg$, then in the far field with $\alpha_g \approx m_C/\kappa \approx 3.270x10^{-59}$, gives a mass

$$m_i = m_p\sqrt{\alpha_g} \approx 1.79\times10^{-37}\ kg\ , \tag{27}$$

which is on the order of the neutrino mass $\left(\sim 10^{-37}\ kg\right)$ and near the earth with $\alpha_g \approx m_C/\kappa \approx 6.782x10^{-39}$, gives a mass

$$m_i = m_p\sqrt{\alpha_g} \approx 1.79\times10^{-27}\ kg\ , \tag{28}$$

which is on the order of the proton mass $\left(1.67\times10^{-27}\ kg\right)$.

4.2 Cosmological Constant Hierarchy

The author recently noted that the Hubble constant

$$H_0 \approx \left(\frac{1}{3}c^3\Lambda\sqrt{\frac{R_U}{Gm_U}}\right)^{1/2} \approx 2.30\times10^{-18}\,s\ , \tag{29}$$

which gives the cosmological constant

$$\Lambda \approx \frac{3H_0^2}{c^3}\sqrt{G\frac{m_U}{R_U}} \approx 1.29\times10^{52}\ m^{-2}\ . \tag{30}$$

The speed of light c and the Hubble constant H_0 are well defined constants of nature. The Universe mass $m_U \approx 7.54\times10^{53}\ kg$ and the Universe radius $R_U \approx 1.049\times10^{27}\ m$ are a bit variable, but one can assert that the ratio m_U/R_U is fairly constant. Therefore equations (24) and (30) imply that the cosmological constant is a function of the dark matter hierarchy coupling constant β_m' as defined by equation (20). This is shown by combining equation (24) with equation (30), which gives the cosmological constant

$$\Lambda \approx \frac{1}{\sqrt{\beta_m'}}\times\frac{3H_0^2T_{PL}}{c^3}\sqrt{\frac{\beta_c^2}{R}}\sqrt{\frac{m_U}{m_C^3R_U}} \approx 1.29\times10^{52}\ m^{-2}\times\frac{1}{\sqrt{\beta_m'}}\ . \tag{31}$$

This fact could be why the vacuum energy density is not equivalent across gravitational models. For example in classical models, the vacuum energy density

$$\rho_{vac\Lambda} = \frac{c^4}{8\pi}\frac{\Lambda}{G} \approx 6.21\times10^{-10}\ J/m^3\ . \tag{32}$$

Although noting the difference in the value of G and equation (24) and the difference in equations (30) and (31) and by combining with equation (32), gives the vacuum energy density as

$$\rho_{vac\Lambda} \approx \frac{c^4}{8\pi}\frac{\Lambda}{G}\cdot\frac{\beta'_m}{\sqrt{\beta'_m}} \approx 6.21\times10^{-10}\ J/m^3 \times \sqrt{\beta'_m}\ ; \tag{33}$$

indicating that the vacuum energy density is also a function of the dark matter hierarchy coupling constant β'_m as defined by equation (20). That is, the vacuum energy density cannot be accurately defined without first defining the local dark matter hierarchy coupling constant. Noting that in classical models $\beta'_m = \beta'_c = 1$, where in other models there is the possibility that $\beta'_m \neq \beta'_c$.

5. OTHER CHANGE TO THE CLASSICAL GRAVITATIONAL MODEL

Changes in the Chameleon particle mass m_C about an object could also be given using equation (23) as

$$\delta m_C = \left(\frac{\beta'_m}{\left(\beta'_m \pm \delta\beta'_m\right)^2}\times m_c^3\right)^{1/3} \tag{34}$$

where δ indicates a change to the external or internal density field associated with an object that produces a corresponding change δ to the hierarchy coupling constant β'_m.

Changing the hierarchy coupling constant β'_m does not necessarily provide for new forces as the gravitational constant remains the same in each density field. However, since the density fields in and about an object do not coupling directly, but through a thin-shell (see the thin shell mechanism in [1, 2]). It is reasonable to assume that during some phenomena, like symmetry breaking [*e.g.*, 18], there will be time retardation between the internal and external density fields that could lead to new forces. That is, there could exist for short periods of time different values of the Chameleon particle mass m_C across an object. In a simple linear 2-dimension form, such a change can be represented by

$$\Delta m_C = \frac{1}{2}m_C + \frac{1}{2}\delta m_C = \frac{1}{2}\left(\frac{m_c^3}{\beta'_m}\right)^{1/3}\left(1+\left(\frac{\beta'_m}{\beta'_m \pm \delta\beta'_m}\right)^{2/3}\right) \leq m_c \tag{35}$$

where $\Delta m_C = m_C = \left(m_c^3/\beta'_m\right)^{1/3}$ when $\delta\beta'_m = 0$. It is noted that systems under time retardation can be quite complex, whereby equation (28) is just one simple variation.

Equation (35) leads to changes in the Gravitational constant. That is, it may be possible even for experiments on the earth to change the coupling to local dark matter by varying the internal or external densities about an object, where a change Δ in the gravitation constant G arises when

$$\Delta G = \frac{1}{\left(\Delta m_C\right)^3}\left(\frac{\beta_c^2}{\beta'_m}\right)\frac{T_{PL}^2}{R} = 8\left(1+\left(\frac{\beta'_m}{\beta'_m \pm \delta\beta'_m}\right)^{2/3}\right)^{-3} G = \theta G \tag{36}$$

where

$$\theta = 8\left(1+\left(\frac{\beta_m'}{\beta_m' \pm \delta\beta_m'}\right)^{2/3}\right)^{-3}. \tag{37}$$

Such that, for example, the gravitation constant G in classical gravitational models must be replaced by

$$G' = \frac{1}{2}(1\pm\theta)G, \tag{38}$$

where the subscript ' implies a variation to the classical gravitational model.

Since the classical gravitation force $F = mg = GMm/R^2$, equation (38) implies a fifth force gravitational model, which is typically given in the form $F = (1+\theta)mg$, where θ is defined as the fifth force coefficient. The variation to the classical gravitational force is then given as

$$F' = m\left(\frac{G'M}{R^2}\right) = mg', \tag{39}$$

where g' implies that the gravitation constant G is not a fixed constant in classical gravitational models.

6. RELATION TO QUANTUM GRAVITY

Quantum gravity, broadly construed, is a physical theory incorporating both the principles of general relativity and quantum theory, which is still ‘under construction. Part of the problem is a still-unanswered question: Although approaches have leads to quantum formulation for gravity, researchers are still busy trying to work out how our universe – which obeys the classical (*i.e.* non-quantum) general theory of relativity – can arise from such a quantum foundation.

In earlier research on quantum gravity it was often supposed that if there was at least one quantum field in the world together with the gravitational field, then given the universal coupling of the gravitational field, it must follow that the quantization of the one field somehow infects the gravitational field, implying that it must necessarily have quantum properties too.

Note, however, that this does not mean that the project of quantum gravity itself rests on unsteady ground: if there are quantum fields and gravitational fields in the world, then given the nature of gravity, we need to say *something* about the manner in which they interact. What is being questioned is whether this means that gravity cannot itself remain fundamentally classical while interacting with quantum fields. After all, as far as all our experiments show: gravity is classical and matter is quantum.

If it is to remain fundamentally classical, then there is the simple question of what such a classical gravitational field would couple to: the quantum properties?

Whether or not spacetime is discrete, the quantization of spacetime entails that our ordinary notion of the physical world that of matter distributed in space and time is at best an approximation. This in turn implies that ordinary quantum theory, in which one calculates probabilities for events to occur in a given world, is inadequate as a fundamental theory. The problem is that our fundamental theories of *matter and energy*, the theories describing the interactions of various particles via the electromagnetic force and the strong and weak nuclear forces, are all *quantum* theories. In quantum theories, these physical quantities do not in general *have* definite values. The hierarchy of density fields presented here asserts a similar scenario, where certain classical gravitational and cosmological constants are dependent on the local field density, which is subject to variations through matter fluctuations within the fields. This could allow a different path in quantum gravity.

For example, quantum field theory general requires a fixed background in order to localize quantum fields and set up causal structure. However, Cao [19] notes that a relational account of localization could perform such a function, with fields localized relative to each other. Such a relation of localization exist between adjacent density fields; allowing for a synthesized entity (*i.e.*, the Universe) of violently fluctuating, universally coupled quantum gravitational (density) fields, which should be what a quantum theory of gravity ought to describe.

All approaches to the problem of quantum gravity agree that something must be said about the relationship between gravitation and quantized matter. These various approaches can be catalogued in various ways, depending on the relative weight assigned to general relativity and quantum field theory. The hierarchy model presented here provides such a link between gravitation and quantized matter through the fact that the matter in a given density field – defines that field, matter which is quantized in nature and provides a link to fluctuations of the density field.

6.1 Planck Scales and Quantum Gravity

Theories of quantum gravity are expected to be able to provide a satisfactory description of the microstructure of spacetime at the so-called Planck scale, at which all fundamental constants of the ingredient theories, c (the velocity of light in vacuum), $\hbar$ (Planck's constant), and G (Newton's constant), come together to form units of mass, length, and time. This scale is so remote from current experimental capabilities that the empirical testing of quantum gravity proposals along standard lines is rendered near-impossible.

However, it is almost gospel that quantum gravity is what happens when you reach the Planck scale. The standard refrain is that 'something peculiar' happens to our concepts of space, time, and causality at such scales requiring radical revisions that must be described by the quantum theory of gravity. The scales at which theories make their mark are set by the values of the fundamental constants. In this way the constants demarcate the domains of applicability of theories: c tells us when specially relativistic effects will become apparent, $\hbar$ tells us when quantum effects will become apparent, and G tells us when gravitational effects will become apparent.

These units $(c, G, \hbar)$ are chosen in such a way to achieve units of length, mass, time and density:

$$l_{pl} = \sqrt{\frac{hG}{c^3}} \approx 10^{-35} m \ ;$$
$$m_{pl} = \sqrt{\frac{hc}{G}} \approx 10^{-8} kg \ ;$$
$$t_{pl} = \sqrt{\frac{hG}{c^5}} \approx 10^{-43} s \ ;$$
$$r_{pl} = \frac{c^5}{hG^2} \approx 10^{93}\ kg/m.$$

Generally, the hierarchy of density dependent field model steps outside the framework of quantum gravity by presenting a more fundamental, quantum theory, which can be investigated under circumstances where one, gets something that looks like a classical spacetime, but allows for gravitational field variations related to fluctuations in the local density field. Therefore, it seems fitting to fixate on the gravitational constant G as implied in this paper, where using equation (24) allows the Planck units to be dependent on the far density field implied by the hierarchy coupling constant according to

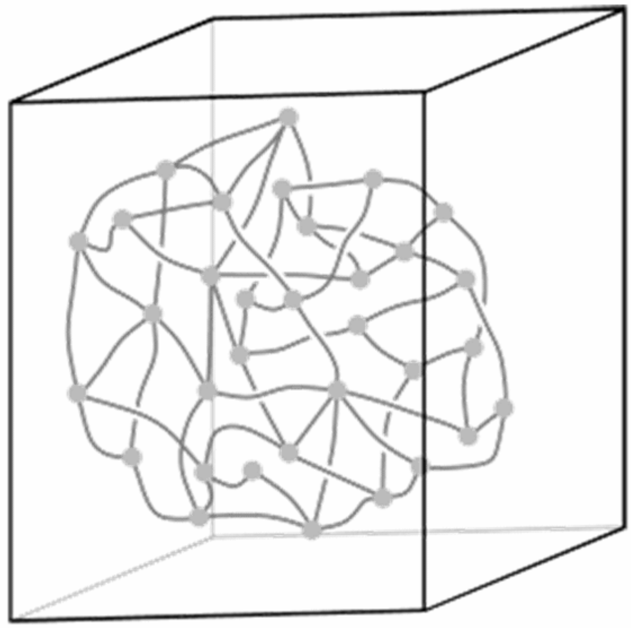
$$l_{pl} = \sqrt{\frac{hG}{\beta_c' c^3}} \approx \frac{10^{-35} m}{\sqrt{\beta_c'}} = 10^{-35} m \ ;$$
$$m_{pl} = \sqrt{\beta_c' \frac{hc}{G}} \approx \sqrt{\beta_c'} \times 10^{-8} kg = 10^{-8} kg \ ;$$
$$t_{pl} = \sqrt{\beta_c' \frac{hG}{c^5}} \approx \sqrt{\beta_c'} \times 10^{-43} s = 10^{-43} s \ ;$$
$$r_{pl} = \left(\beta_c'\right)^2 \frac{c^5}{hG^2} \approx \left(\beta_c'\right)^2 \times 10^{93}\ kg/m = 10^{93}\ kg/m,$$

where $\beta_c' = 1$.

However, under certain - yet definable - conditions were the density field fluctuates, β_c' maybe replaced by β_m' or $\delta\beta_c'$, allowing even these units to change; imposing changes at the Planck scale that could propagate to our scale, *i.e.*, quantum gravity.

6.2 Loop Quantum Gravity

Loop quantum gravity is an attempt to develop a quantum theory of gravity based directly on Einstein´s geometrical formulation. This approach is hard to explain without resorting to the elusive language of mathematics. Although, o ne aspect of the loop models, though, is easy to grasp. Space, in general relativity, is a continuum. In every part of it, one can define regions of arbitrarily small volume, and every little region can be divided further into yet smaller regions, ad infinitum. In the loop models, the basic structure of space-time turns out to be discrete. In such discrete space-times, there are smallest values for volumes and areas that are not divisible any farther – just as one cannot build a structure smaller than the smallest block in a children's lego set. The fabric of space is called a *spin network* with lines and nodes, as pictured here:



Nodes can carry numerical values; depending on the number, they stand for volume building blocks of different size. The smallest possible volume is that of a region containing only a single node with the lowest possible value. As you add further nodes and/or make the values associated with existing nodes larger, the volume grows.

Thus, space acquires a grainy, discrete structure – and so does time. In simplified models used for cosmological explorations, it turns out that within loop quantum gravity, there is no big bang singularity; instead, the universe´s history can be traced infinitely far into the past, step by step.

The hierarchy of density dependent field model related to the loop model, in that, it also defines the basic structure of space-time to be discrete. That is, one can define the smallest values for volumes as having a hierarchy coupling constant, where $\beta_c' = 1$ and not divisible any farther.

Further, from a quantize prospective, it is asserted that changes in the Chameleon particle mass m_C about a given mass could be quantized by association. Whereby, using equation (34), the Chameleon particle mass change

$$\delta m_C = \left(\frac{\beta_m'}{\left(\beta_m' \pm \delta\beta_m'\right)^2} \times m_c^3 \right)^{1/3} = n\left(\frac{m_c^3}{\beta_n'} \right)^{1/3} \tag{40}$$

where n is the respective quantum numbers related to the number of dark matter particle mass m_c present in a given density field ϕ_n and where β_n' is not necessarily equally to β_m', as it may need to be somehow quantum related to the mass in question. However, in far space

$$\delta m_C \approx n\left(\frac{m_c^3}{n\beta_c'} \right)^{1/3} = \left(n^2 m_c^3\right)^{1/3} \tag{41}$$

as there are only dark matter particle masses to speak of. Equation (41), implies that a given mass $m_i = \delta m_C$ cannot exist in a given density field until

$$\beta'_m = \frac{1}{n^2} \approx \left(\frac{m_c}{m_i} \right)^3 \quad (42)$$

which quantizes the inverse of the dark matter coupling constant β'_m [and therefore the mass coupling constant β_m per equation (20)] to specific matter species. However, it would then be more likely that

$$\beta'_m = \left(\sum_{k=1}^{j} n_k \right)^{-2} \approx \left(\frac{m_c}{\sum_{k=1}^{j} m_k} \right)^3 \quad (43)$$

where m_k denotes a mass species existing in a given density field and n_k is the number of the mass species m_k in the density field.

7. GENERAL DISCUSSION

In this paper, Chameleon Cosmology [1, 2], which describes an additional force to gravity, is expanded to include effects caused at the microscopic level of the Chameleon particle mass m_C, which is equivalent to the dark matter particulate mass m_c in far-space, but increases in denser density environments as near the earth due to a dark matter hierarchy coupling constant. This picture differs from classical gravitational field models, in which distant sources produce varying potential fields. However, since under normal conditions this additive force is much-much smaller than normally derived gravitational forces, this model adds no significant change to classical gravitational models.

Since the Chameleon particle mass m_C occupies the same topography as all other gravitational models, it shares the same origins in the basic structure of the universe; providing similarities down to the laboratory scale through fundamental hierarchy coupling. This similarity was shown to be connected to the gravitational constant G and could explain why a true fundamental value of the gravitational constant has yet to be experimentally defined [20]. Whereby a better understanding of how to control such coupling in local environments could lead to new force produce devices outside known gravitational models while being based on astrological observations.

8. FINAL REMARKS

Connections to physical constants in classical gravitational models evolving new physics as dark matter and the derivation of a density dependent hierarchy of scalar fields in the universe provides a starting point for extension from cosmology and astrophysics to other physical fields as particle and solid state physics within the earth's density field, which could provide foundations for modifying old or developing new quantum gravity theories.

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