Engineering Dynamics of a Scalar Universe
Part I: Theory & Static Density Models

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Abstract. The notion that our universe is composed of scalar fields is becoming more of a fact as we learn more about the nature of the universe. The most appealing fact toward this is the discovery that the cosmos is expanding due to vacuum or dark energy. Cosmological expansion presents itself as a fifth force. In this paper, fifth force models are developed base on the Chameleon scalar field model presented by Khoury and Weltman that presents an alternate means of acquiring the same force results as attainable from the standard Newtonian force models. These models are incomplete, but with further development, could lead others to develop force producing devices using unforeseen methods not visible under our current models.

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INTRODUCTION

There is a growing belief that gravitational forces are rooted in quantum field theories. However, the most prominent governing gravitational theory, General Relativity fails to be quantized; rather it appears to map (spacetime) the quantization effects rather than defining it. As a result, only the effects of quantization can be predicted by General Relativity. Unfortunately, this can lead to false theories, which lack the ability to be defined by quantum field theories - on the large scale. This includes oddities as Worm Holes and the like that in some form requires exotic matter or energy to be achieved.

On the other hand, when quantization is applied to spacetime, the resulting theories derive energy states; the most common theory being known as zero point energy, which is a form of residual electromagnetic energy in the universe and is sometime referred to as exotic. However, these subscale energy states are detached from the large scale matter states of the universe; posing very little potential for predicting gravitational effects, which could lead to new gravity like interaction.

We are thus left with a conundrum; one that produces a discontinuous line from the small scale to the large scale.

In 2004, a new theory was proposed, a theory that indirectly proposed a quantum connection to the large scale effects of gravity. This theory is called the Chameleon Theory (Khoury and Weltman, 2004a and 2004b) as it poses to hide within know gravitational physics through its connection to the density of the surrounding matter field densities. Plus the calculated effects only add a very-very small addition to gravity, so small it can be neglected in most all analysis.

In this paper, the Chameleon model is broadened into a generalized force model through a concept known as the fifth force search; a technique to compare new cosmological forces to gravity. In doing so, the Chameleon model is shown to extend to other acceleration models, where forces are much greater than those proposed by Khoury and Weltman (2004a and 2004b).

Particular to this forum, the integration of the fifth force search model and the Chameleon model produces a new engineering force model (referred to as a local fifth force model) that can be used for all acceleration and momentum exchange models on the large scale. This model could lead to new developments in propulsion engineering (see; Robertson, 2009) and possibly new power producing devices.

The author notes that the some of the models developed in this paper are incomplete, but are presented to show that alternate means of acquiring the same force results are achievable using this new engineering force model.

BACKGROUND

There is widespread interest in the possibility that, in addition to the matter described by the standard model of particle physics, the vacuum of our Universe may be populated by one or more scalar fields, which are generally featured in high energy physics and embedded in quantum field theory. In cosmology the existence of vacuum scalar fields has been postulated as a means to explain the early and late time acceleration of the Universe and have given rise to the notion of a fifth force (Mota and Shaw, 2006). A natural question then arises: if such a nearly massless vacuum scalar field exists, why have we not detected it in local tests of either the Equivalence Principle (EP) and/or fifth force searches (Will, 1993 and 2001; Fischbach and Talmadge, 1999)?

Before we can answer this question, the context to which one can understand vacuum scalar fields in the Universe is required. Unfortunately, there are many models with quite different properties in literature. However, the following three models (and subsets) stand out:

Friedmann-Lemaître-Robertson-Walker (FLRW) - the scalar field is the same over the entire universe (see; d'Inverno, 1992). The key feature of FLRW cosmology is that it is a simplification of the Einstein equation where you assume the universe is homogeneous with an isotropic scalar field, which allows them to be analyzed in some detail.

Arbey - the scalar field is attractive (dark matter) on a galactic scale, but repulsive (dark energy) on a cosmological scale (Arbey, 2006).

Khoury and Weltman - the Chameleon scalar field is smooth on a cosmological scale, but has huge gradients on the scale of laboratory-sized objects (Khoury and Weltman, 2004a and 2004b) and is a dark energy model.

In the following, the Friedmann-Lemaître-Robertson-Walker (FLRW) model is built upon by first analyzing a miniature FLRW universe presented with factors of $\hbar$ and c, which are appropriate for scalar fields expressed in mass units, such as grams or k-grams. (These factors are also used throughout this text for clarification of engineering units.) The FLRW model is then converted to an expansion model, which allow for the introduction of a fifth force coefficient, which is simply the ratio of the acceleratory or vacuum force to the Newtonian force. This is followed by a discussion on the Chameleon model (Khoury and Weltman, 2004a and 2004b), which allows for the development and analysis of several local fifth force models with respect to the change in the fifth force coefficients about a mass; the last model being a new momentum exchange model.

THE FRIEDMANN-LEMAÎTRE-ROBERTSON-WALKER SCALAR UNIVERSE

In the FLRW universe, the acceleration of matter is uniform at every point as to produce isotropic velocity vectors when viewed from any object. That is, no matter which direction we look into the night sky, the distribution of galaxies is pretty much the same. This feature of the Universe is known as "isotropy" and hides the true center of the Universe. Instead it is required to define a position $x(r)_{\min}$ - defined here as the baryonic center and having a dimensional length set to zero - toward which an the gravitation force of all galaxies points. This feature also hides the outer edge of the Universe – define here as the expansion horizon and having a dimensional length $x(r)_{\max} = R_{Ex}$, which may or may not be equivalent to the current radius R_0 of the Universe.

The acceleration of the Universe is commonly thought of as being caused by a vacuum pressure and due to the "now standard" notion of dark energy as implied by models like that of Arbey (2006) and Khoury and Weltman (2004a and 2004b). As such, a Friedmann Scalar Universe can be described in a 2D or linear prospective of vacuum pressure as is given in Figure 1. This 2D model implies that the mass m sees the universe expanding informally away from its position in space.

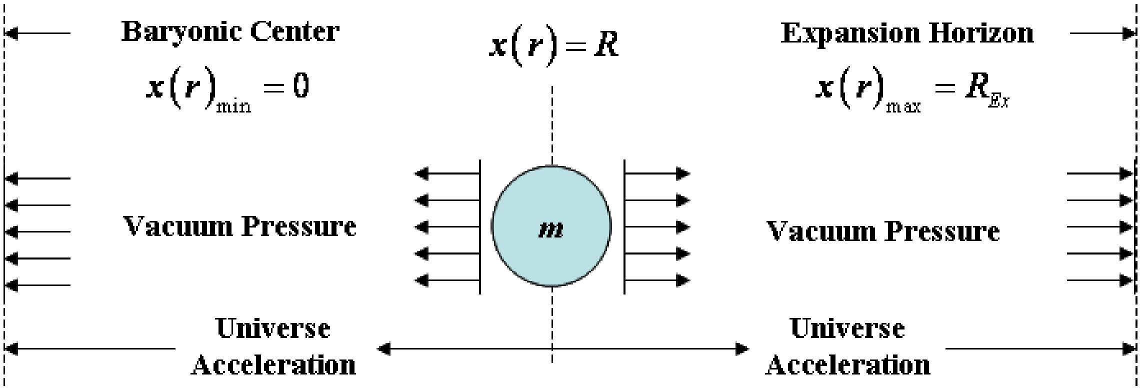

FIGURE 1. 2D FLWR Scalar Universe Model

Miniature Universe Model

To understand the dynamics of the FLWR Scalar Universe, a miniature spherical universe - to include various radiations - of mass M and radius $x(r) = R$ is constructed consisting of a large spherical region, which is cut out of our greater universe; containing a large number or super-cluster of galaxies. Allowing the mass m in Figure1 to be a galaxy placed at the border, where $x(r) = R$ of this miniature universe, the galaxy will have a matter potential energy E_p^M determined by the total mass m included in the miniature universe and concentrated about its baryonic center $x(r) = 0$, given by

$$E_p^M = -m\left(MG_N/R\right) \tag{1}$$

where the total matter density ρ_M of the miniature universe is given by

$$\rho_M = 3M/4\pi R^3 \ . \tag{2}$$

The vacuum energy carries the normally ascribed vacuum energy density

$$\rho_v = \Lambda c^4/8\pi G_N \ , \tag{3}$$

which indicates that the cosmological constant Λ and the vacuum energy density ρ_v are proportionality constants. That is, $\Lambda/\rho_v = 8\pi G_N/c^4$ is constant, where over time as the vacuum energy density ρ_v (or dark energy in the universe) changes, so does the value of the cosmological constant.

The vacuum energy then possesses a vacuum potential energy

$$E_p^v = -\rho_v\left(4\pi R^3/3\right) = -1/6\left(\Lambda c^4/G_N\right)R^3 \ . \tag{4}$$

Total Potential Energy

The "total energy" is determined by allowing the miniature universe to expand or contract, where the total energy E_{tot} of the border galaxy will become the sum of the kinetic energy $m\left(dR/dt\right)^2/2 = m\dot{R}^2/2$ and the potential energies, such that, the total energy becomes

$$E_{tot} = E_p^M + E_p^v + m\dot{R}^2/2 \tag{5}$$

or by combining with equations (1) and (4) to obtain

$$E_{tot} = -m\left(G_N M/R - \dot{R}^2/2\right) - 1/6\left(\Lambda c^4 R^3/G_N\right). \tag{6}$$

Hubble Expansion

The Hubble parameter H varies with time but is the same everywhere in the universe at a given time counted from the Big Bang. (As of this writing, the current measured value for the Hubble parameter $H \equiv H_0 \approx 71\ km\ s^{-1} Mpc^{-1} (\approx 2.30 \times 10^{-18}\ s^{-2})$. Therefore, the dynamics of space in the miniature universe must also include the Hubble expansion H, which relates the recessional velocity $v = HD$ of two cosmological objects to their relative distance D, and is introduced into equation (6) by defining

$$\dot{R} \equiv HR. \tag{7}$$

To give the total energy as

$$E_{tot} \approx -m\left(G_N M/R - H^2 R^2/2\right) - 1/6\left(\Lambda c^4 R^3/G_N\right). \tag{8}$$

Now the geometry of space in this miniature universe must be specified by solving Einstein's equation $R_{\mu\nu} - g_{\mu\nu}R/2 = -8\pi T_{\mu\nu}/c^2 + \Lambda g_{\mu\nu}$, which relates mass-energy to the curvature of space and shows the influence of the cosmological constant. To make a long story short, the result is simply

$$E_{tot} = -k \cdot mc^2/2 \tag{9}$$

with $k = 0$ for a flat universe with an Euclidean geometry, $k = +1$, for a universe with a positive curvature such as a sphere, $k = -1$ for a hyperbolical geometry that represents the topological surface of a saddle.

Then setting equations (8) and (9) equivalent and dividing through by the galaxy mass m,

$$H^2 = 2G_N M/R^3 + 1/3\left(\Lambda c^4 R/mG_N\right) - kc^2/R^2. \tag{10}$$

Noting that when the total mass of the universe is taken into account with $m = M \approx \left(c^2/G_N\right)R$, the Friedmann equation (Friedmann, 1922) $H^2 = \left(2G_N M/R^3 + 1/3\Lambda c^2 - kc^2/R^2\right)$ is obtained.

Critical Density

By letting $k = 0$ in equation (10), which corresponds to a flat Euclidian universe as proven by the Wilkinson Microwave Anisotropy Probe (WMAP) data (see; http://map.gsfc.nasa.gov), the critical density ρ_c or the sum of all matter, radiation and dark energy in the universe is determined as

$$\rho_c \equiv \rho_M + \rho_v/c^2 = 3H^2/8\pi G_N \tag{11}$$

The current estimate of the critical density $\rho_c \approx 10^{-26}\ kg/m^3$.

Force

Since force is defined as the negative of the gradient of the potential, the force F on the border galaxy is given as

$$\vec{F} = -\left(dE_p^M/dr + dE_p^v/dr\right)\hat{r} = -m\left(G_N M/R^2\right)\hat{r} + 1/3\left(\Lambda c^4 R^2/G_N\right)\hat{r}, \tag{12}$$

where equation (12) shows that the matter distribution inside our universe has a gravitational attraction pointing toward its baryonic center and an acceleratory force from the vacuum energy distribution. This gives the usual attractive Newtonian gravitational force

$$\vec{F}_N = m\left(-G_N M/R^2\right)\hat{r} \equiv m\vec{g} \tag{13}$$

and the vacuum force acting opposite to the direction of the baryonic center as

$$\vec{F}_v = 1/3\left(\Lambda c^4 R^2/G_N\right)\hat{r} \approx 1/3\left(\Lambda c^2 R \cdot M\right)\hat{r} \equiv M\ddot{R}\ \hat{r}. \tag{14}$$

Equation (14) indicates that the vacuum force is directly proportional to the mass M in the Universe times the universe acceleration factor $\ddot{R} \equiv \Lambda c^2 R/3$.

Vacuum Pressure

In addition to the Friedmann equation (10), another key equation of cosmology involves the universe acceleration factor $\ddot{R}$, given by

$$\ddot{R}/R \approx -4\pi G_N/3\left(\rho_c + 3p/c^2\right) + \Lambda c^2/3\,, \tag{15}$$

where p is the vacuum pressure responsible for cosmological expansion and $3p/c^2$ is the pressure energy density. Given the universe acceleration factor $\ddot{R}$ from equation (14), it is easily seen that equation (15) yields

$$\rho_c \approx -3p/c^2$$

(16)

and indicates that vacuum pressure p producing the observed accelerated expansion of the universe is negative – directed away from the baryonic center . Then using equations (2), (7) and (11) the vacuum pressure $p \approx -1/3\left(\rho_M c^2 + \rho_v\right) \approx -\rho_M \dot{R}^2/6$, which for $\rho_M c^2 >> \rho_v$, defines the speed of light in a vacuum as $c \approx \dot{R}/\sqrt{2} = HR/\sqrt{2}$; indicating that the speed of light in a vacuum is would be unchanged until $\rho_M c^2 \to \rho_v$.

THE FIFTH FORCE COSMOLOGICAL EXPANSION MODEL

Fifth force searches look for changes in the local Newtonian force F_N according to

$$\vec{F} = (1 \pm \theta_{L_i})\vec{F}_N\,, \tag{17}$$

where here we define θ_{L_i} as the local fifth force coefficients associated with Newtonian mass that effects the total force F on any other near by mass; denoted by the subscript $\left({}_i\right)$. In consideration of cosmological expansion, the dark energy in the universe provides such a fifth force coefficient term, such that, cosmological expansion can be given by the fifth force coefficient θ_{Exp}, yielding the total force

$$\vec{F} = (1 - \theta_{Exp})\vec{F}_N\,, \tag{18}$$

where the subscript $\left({}_{Exp}\right)$ denotes the cosmological expansion and the cosmological Newtonian force $\vec{F}_N$ defined by equation (13) .

Now by extending the previous miniature universe analysis to equation (18) for the total baryonic (plus radiation) mass M with radius R and equating it to equations (12-14) gives the expansion force as

$$\vec{F} = (1 - \theta_{Exp})\vec{F}_N = \left(\vec{F}_N + \vec{F}_v\right), \tag{19}$$

such that, the cosmological expansion fifth force coefficient becomes

$$\theta_{Exp} = -\vec{F}_v/\vec{F}_N\,, \tag{20}$$

where the negative sign comes from the direction of the Newtonian force $\vec{F}_N$.

The Fifth Force Expansion Model

The expansion fifth force coefficient θ_{Exp} of equation (20) implies that the universe is expanding. However, in the 2D FLWR Scalar Universe Model of Figure 1, the galactic mass m sees a uniform acceleration of all objects away from it as if is motionless. Therefore, the 2D FLWR Scalar Universe Model of Figure 1 must be modified to the 2D Cosmological Expansion Model as shown in Figure 2, where the vacuum pressure is taken to be greatest at $x = 0$

and weakest at $x = R_{Ex}$ and in like to Figure 1, the mass m_G is a galaxy with radius R_G positioned at the edge of a miniature universe with mass M and radius $R >> R_G$.

Although the 2D Cosmological Expansion Model is anisotropic and not isotropic in velocity, in the following it is shown that this model gives the same cosmological acceleration force as the 2D FLWR Scalar Universe Model.

First it is noted that the cosmological acceleration force F_v on the galactic mass m_G can be given in terms of the dark energy density ρ_v of equation (3) by combining it with equation (14) to yield

$$\vec{F}_v = \left(8\pi\rho_v x^2/3\right)\hat{x} , \tag{21}$$

where the value of x varies from the surface of the mass m_G with respect to the mass's center with an associate variation in the vacuum pressure about the mass in the direction of motion.

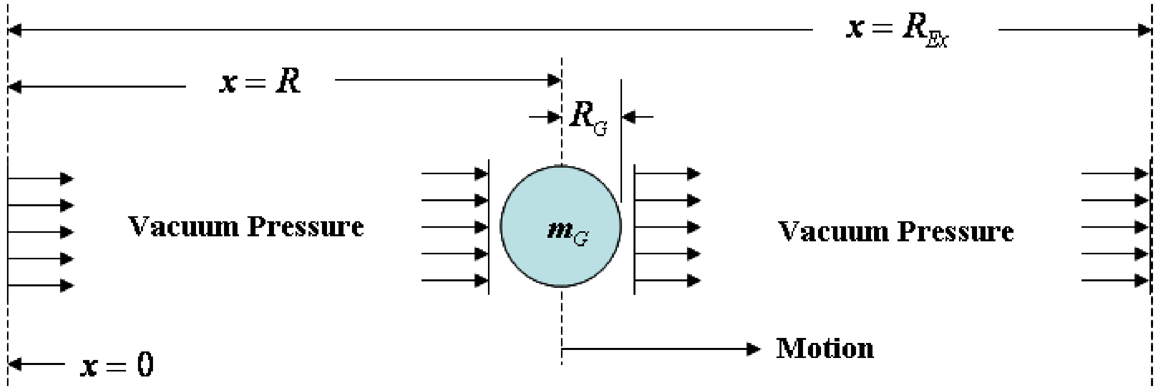

FIGURE 2. 2D Cosmological Expansion Model

In a similar manner, the local Newtonian force F_N that is directed opposite to the direction of expansion can be given in terms of the miniature universe's matter density ρ_M of equation (2) by combining it with equation (13) as

$$\vec{F}_N \approx -m_G\left(4\pi G_N R\rho_M/3\right)\hat{x} . \tag{22}$$

The Cosmological Expansion Fifth Force Coefficient

Now combining equations (21) and (22) with equation (20) gives the cosmological expansion fifth force coefficient θ_{Exp} about the galactic mass as

$$\theta_{Exp} = -\vec{F}_v/\vec{F}_N = \left(2x^2/m_G G_N R\right)\left(\rho_v/\rho_M\right) . \tag{23}$$

where the position x must be evaluated about the galactic mass m_G and implies (from the following analysis) using the sum of the local expansion fifth force coefficients on either side of the galactic mass m_G , where

$$\theta_{Exp} = \sum\theta_{L_m} \equiv 1/2\left(\theta_{L_m}\right)_{left} + 1/2\left(\theta_{L_m}\right)_{right} \equiv 1/2\left(\theta_{L_m}\right)_{x=R-R_G} + 1/2\left(\theta_{L_m}\right)_{x=R+R_G} , \tag{24}$$

where the factor of ½ comes from the evaluation of the fifth force coefficient or force across the galactic mass.

From equation (17) and using equations (19) and (23), equation (24) gives the net expansion force $\vec{F}_v$ on the galaxy as

$$\begin{aligned}\vec{F}_v &= -\theta_{Exp}\vec{F}_N = -\left(\left(R-R_G\right)^2/R + \left(R+R_G\right)^2/R\right)\left(1/mG_N\right)\left(\rho_v/\rho_M\right)\vec{F}_N \\ &= -2\left(R + R_G{}^2/R\right)\left(1/mG_N\right)\left(\rho_v/\rho_M\right)\vec{F}_N\end{aligned} .$$

However, since $R_G << R$, this equation can be reduced to

$$\vec{F}_v \approx -\left(2R/mG_N\right)\left(\rho_v/\rho_M\right)\vec{F}_N \,, \tag{25}$$

which can be shown equivalent to the expansion force F_v given by equation (14) of the FLWR model; indicating that these two model are equivalent.

Whereby, the cosmological expansion fifth force coefficient θ_{Exp} for a given mass m in our Universe of radius R_0 can be given as

$$\theta_{Exp} = -\vec{F}_v/\vec{F}_N \approx 2\left(R_0/mG_N\right)\left(\rho_v/\rho_M\right) . \tag{26}$$

> **Example 1** – For the earth mass $m_{\oplus} = 5.97x10^{24}\ kg$ with $\rho_v = 8.17x10^{-44}\ J/m^3$, $\rho_M = 2.7x10^{-27}\ kg/m^3$ and $R_0 \approx 1.03x10^{27}\ m$, the Cosmological Expansion fifth force coefficient $\theta_{Exp_{\oplus}} \approx 1.57x10^{-4}$. Therefore if the earth was not bound by the gravitational mass of the galaxy, in one million years the earth's acceleration would increase by a factor of 157. However, since the mass of the Milky Way galaxy is much bigger than the earth mass, $\theta_{Exp_{MW}} \approx 4.25x10^{-21}$ which indicates a much slower expansion rate.

THE CHAMELEON MODEL

A novel scenario offering a natural resolution to the conflict of local fifth force searches was proposed by Khoury and Weltman (2004a and 2004b) with further work by Brax *et al* (2004a and 2004b). Their work proposes a scalar field which can evolve on a Hubble time today and cause cosmological acceleration, while coupling to matter with gravitational strength. The basic idea is that the scalar field acquires a mass which depends on the local background matter density. On Earth, where the density is high, the Compton wavelength of the field is sufficiently short to satisfy all existing tests of gravity. In the solar system, where the density is many orders of magnitude smaller, the Compton wavelength is larger than the size of the solar system while in the cosmos, where the density is tiny, the field can have a mass of order of the present Hubble constant and cause cosmic acceleration. Because it's physical properties depend sensitively on the ambient or surrounding environment (i.e., mass density), this scalar field is dubbed the "Chameleon."

The Chameleon Model is basically a density-dependent mass model and while the idea is not new (Wetterich, 1995; Anderson and Carrol, 1997; Damour and Polyakov, 1994a and 1994b; Huey *et al*, 2000; Hill and Ross, 1988; Ellis *et al*, 1989), their model is novel in that the Chameleon's scalar field, like dark matter (Brax *et. al*., 2007) can couple directly to "baryons with weak gravitational strength." Therefore, gravitational coupling to baryons makes this model of interest toward the understanding of new momentum systems.

Quantum Oscillations and Time Dependent Field Momentum

The ambient or external Chameleon field (the local scalar force medium) must have time dependency to store field momentum in order to affect the gravitational field near a mass. Several textbooks (Ramond, 1981) relate the scalar field momentum density P (Low, 1997) to the product of the time derivative ∂ and gradient ∇ of a scalar field $\phi(x,t)$ (given as a function of distance x from an object and the perturbation time t) according to $P = -\int \partial^3 x \cdot \partial\phi(x,t)/\partial t \times \nabla\phi(x,t)\left(c/\hbar\right)$.

To give time dependence to the Chameleon model, the author noticed that the Chameleon model deals with baryonic matter down to the Planck length scale through the model's dependence on the cosmological energy scale factor

$$M_E \approx \left(\Lambda/8\pi l_p^2\right)^{\frac{1}{4}} \approx 10^4\ m^{-1} ; \tag{27}$$

providing a connection between the large scale – cosmological constant Λ, which deals with cosmological expansion (dark energy) and the very small scale – Planck length l_p, which deals with quantum phenomena.

The author suggests that this quantum connection comes about in a second effect in the Chameleon model called the thin-shell mechanism (discussed in more detail in the next section), which is a very thin energy layer about an object that suppresses the mass contribution to the ambient Chameleon field. The quantum connection of equation (27) to the Chameleon field (see equation (32)), both internal and external to a mass, then allows the thin-shell to be visualized as sub-atomic matter at the surface of a mass to be in a state of *quantum harmonic oscillation,* which induce perturbations in the ambient Chameleon Field. To compensate for the localized perturbation in the ambient Chameleon field, energy-momentum is produced. In such a case, the spatial momentum would be directed away from the perturbation, appearing as negative gravitational pressure; indicating that the energy of the oscillating mass in the Chameleon field must be a form of dark energy.

The Chameleon Scalar Field

The scalar field properties of the Chameleon field to include the thin-shell, presents a self-interaction potential energy, which infers that the thin-shell has a near constant energy potential density $V_\phi(x=r_m,t)$, where r_m is the radius of the mass and where the time t is in some way proportional to the Planck time to allow correlation with *quantum oscillations*. However, the energy potential density $V_\phi(r,t)$ in the external Chameleon field is a function of the position $r>r_m$ and can change. On the other hand, as the Chameleon model is a mass density model, in the general case where a mass's density is constant, it is assumed that the time t at any position $r>r_m$ is the same as in the thin-shell. Provided it is unaffected by any other mass's external Chameleon field. Plus it is assumed that the time t approaches the Planck time far from any mass, whereby it is expected that the time t is proportional ratio of the mass's density to the Universe's density. (Time varying mass density will be discussed later.)

The cosmological energy scale factor M_E of equation (27) and the energy potential $V_\phi(r,t)$ comes to play in the Chameleon model by the model's focus on the scalar field

$$\phi(r,t)^n=\left(M_E^{4+n}/V_\phi(r,t)\right)\cdot\left(\hbar^3/c\right) \tag{28}$$

where $1<n<4$ to fit theory compatible with cosmological and laboratory data (the favored value is $n=2$), and imposing the boundary conditions so that the solution is non-singular at the origin and that the object's external Chameleon field tends to the external or ambient Chameleon field value $\phi_{C_0}\sim \mathrm{e}^{-m_0 r}/r$ far from the object.

Whereby, the time derivative of equation (28) applied to the scalar field momentum density $P=-\int\partial^3 r\cdot\partial\phi(r,t)/\partial t\times\nabla\phi(r,t)(c/\hbar)$ in the volume about static (non-moving) objects (like the earth) and implies that the objects Chameleon field-momentum is a function of the distance r from the object. On the other hand, non-static (masses in motion or masses with a changing density) objects implies that the objects Chameleon field-momentum is a function of the distance r from the object and the time varying perturbation time $t=\Delta t$, which could lead to greater scalar field momentum densities, as time-vary phenomena typically have a $1/\partial t$ or frequency dependency.

Thin-Shell Mechanism

The thin-shell mechanism was developed by concentrating on the static solution where $t=0$, such that, the external scalar Chameleon field was derived to be

$$\phi(r,t)\equiv\phi(r)\approx -m\left(1/4\pi M_{Pl}\right)\left(3\Delta R_m/R_m\right)\left(e^{-r/\lambda_{C_\infty}}/r\right)\left(\hbar/c\right)+\phi_{C_0} \tag{29}$$

about a spherically symmetric and significantly large object of homogeneous density ρ_m, mass m and radius R_m having a thin-shell thickness ΔR_m. The subscript $(_\infty)$ in equation (29) is to denote of the external Chameleon field far from the object. The original Khoury and Weltman version of equation (29) contains an additional coupling factor term, which they set to 1. Here this coupling factor is combined within ΔR_m; see equation (37).

The Compton wave length or effective range λ_{C_m} of an object's Chameleon field is given by

$$\lambda_{C_m} \approx \left(1/m_{C_m}\right)\left(\hbar/c\right), \tag{30}$$

where an object's internal Chameleon field mass m_{C_m} is given by

$$m_{C_m} = \left(\sqrt{6}/M_E\right)\left(\beta_{C_m}\cdot\rho_m/2M_{Pl}\right)^{2/3}\left(\hbar/c\right). \tag{31}$$

and where $M_{Pl} \equiv \sqrt{\hbar c/8\pi G_N}$ is the reduced Planck mass. The coupling factor β_{C_m} is between an object's internal Chameleon field and in the Chameleon model is ~ 1, which indicates that an object's thin-shell shields the object's mass from the external Chameleon field.

The thin-shell model for the galaxy mass m_G of Figure 2 is shown in Figure 3. As shown, within some distance $R_m < r \le \left(R_m + \Delta R_m + \lambda_{C_m}\right)$, there is a perturbation of the vacuum pressure field. The distances ΔR_m and λ_{C_m} are actually $<<$ than the mass radius R_m. They are shown large only for illustration purposes.

It is noted that the Chameleon model ignores perturbations of the Chameleon field for $r > \left(R_m + \Delta R_m + \lambda_{C_m}\right)$ and assumes that the normal vacuum pressure field resumes un-perturbated. It is assumed from the literature that this is done as a way to avoid discussions of experimentation at distances $r > \left(R_m + \Delta R_m + \lambda_{C_m}\right)$, where for static objects the field-force would be much smaller. In this paper, greater distances are discussed.

The effective internal Chameleon field for $r \le R_m$ of an object (to include the ambient field ϕ_{C_0}) is given by

$$\phi_{C_m} = M_E^2\left(2M_{PL}/\beta_{C_m}\rho_m\right)^{1/3}\left(\hbar/c\right), \tag{32}$$

where the object's thin-shell thickness ΔR_m minimizes the effective field potential $\partial V_\phi\left(r, t=0\right)/\partial\phi + \rho_m e^{\phi_c/M_{Pl}}/M_{Pl} = 0$ and holds everywhere inside the object except within a thin shell of thickness ΔR_m at the surface where the Chameleon field potential rapidly grows to compensate for the higher ambient Chameleon field ϕ_{C_0} about the object.

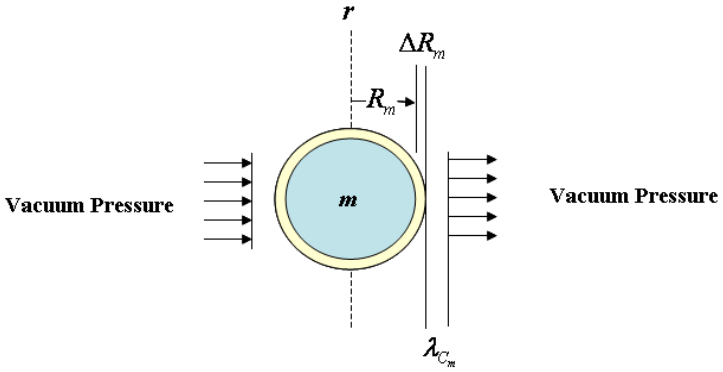

FIGURE 3. Chameleon thin-shell model

Example 2 - For the earth with a density $\rho_\oplus \approx 5520\ kg/m^3$ and $\beta_{C_\oplus} \approx 1$, equation (31) yields an internal field mass $m_{C_\oplus} \approx 5.60\times10^{-39}\ kg$ (the scale of ultra low atomic matter) to give a Compton wave length $\lambda_{C_\oplus} \approx 6.30\times10^{-5}\ m$ with an internal Chameleon field $\phi_{C_\oplus} \approx 5.33\times10^{-39}\ kg$.

For the atmosphere with a density $\rho_{atm} \approx 1.2\ kg/m^3$ and $\beta_{C_{atm}} \approx 1$, equation (32) yields an internal field mass $m_{C_{atm}} \approx 2.03\times10^{-41}\ kg$ (also the scale of ultra low atomic matter) to give a Compton wave length $\lambda_{C_{atm}} \approx 1.74\times10^{-2}\ m$ with an internal Chameleon field $\phi_{C_\oplus} \approx 8.87\times10^{-38}\ kg$.

For the universe with a density $\rho_c \approx 10^{-26}\ kg/m^3$ gives an internal field mass $m_{C_c} \approx 8.33\times10^{-59}\ kg$ (the scale of ultra low Bosonic matter) and $\lambda_{C_c} \approx 4.23\times10^{15}\ m$ with an internal Chameleon field $\phi_{C_c} \approx 8.32\times10^{-31}\ kg$. (It is noted that the universe internal field mass $m_{C_0} \approx 8.33\times10^{-59}\ kg$ $(=4.65\times10^{-23} eV)$ is only a factor ~2.3 higher that the dark matter particle size $\approx 2\times10^{-23}\ eV$ indicated by Silverman and Mallett (2001).)

Thin-Shell Thickness

The thin shell thickness ΔR_m is related to the ambient Chameleon field ϕ_{C_0}, the object of mass m, the internal field ϕ_{C_m} and the Newtonian (gravitational) potential

$$\Phi_N = \left(G_N m/R_m\right)c^{-2} = g_N R_m c^{-2} \tag{33}$$

of the object by

$$3\Delta R_m/R_m = \left(\left(\phi_{C_0}-\phi_{C_m}\right)/6M_{Pl}\right)\left(1/\Phi_N\right) = \left(\left(\phi_{C_0}-\phi_{C_m}\right)/6M_{Pl}\right)\left(c^2/g_N R_m\right), \tag{34}$$

Equation (34) then yields the object's thin-shell thickness

$$\Delta R_m \approx \left(\left(\phi_{C_0}-\phi_{C_m}\right)/18M_{Pl}\right)\left(c^2/g_N\right) = 1/3\left(\left(\phi_{C_0}-\phi_{C_m}\right)/\rho_m R_m\right)M_{PL}\left(c/\hbar\right), \tag{35}$$

or by incorporating equation (30) as

$$\Delta R_m \approx 1/3\left(M_E^2/R_m\rho_m\right)\left(\left(2M_{PL}^4/\beta_{C_0}\rho_0\right)^{1/3} - \left(2M_{PL}^4/\beta_{C_m}\rho_m\right)^{1/3}\right),$$

which shows a strong dependence on the inverse of the ambient field density ρ_0 as it is typically much less the that the object's density ρ_m.

Now since the interface between the ambient field density ρ_0 and the object's density ρ_m produces the object's thin-shell thickness ΔR_m, the Chameleon coupling factors $\beta_{C_0} \approx \beta_{C_m} \equiv \beta_C$ and allows the thin-shell thickness to be defined as

$$\begin{aligned}\Delta R_m &\approx 1/3\left(M_E^2/R_m\hat{\beta}_C\rho_m\right)\left(\left(2M_{PL}^4/\rho_0\right)^{1/3} - \left(2M_{PL}^4/\rho_m\right)^{1/3}\right)\\ &\approx 1/3\left(M_E^2/R_m\hat{\beta}_C\rho_m\right)\left(2M_{PL}^4/\rho_0\right)^{1/3}\left(1-\left(\rho_0/\rho_m\right)^{1/3}\right)\end{aligned}, \tag{36}$$

where $\hat{\beta}_C \approx \left(\beta_C\right)^{1/3}$ is the revised Chameleon coupling factor. [This is shown to be validated for the Earth in Example 4, where equations (35) and (36) produce near equivalent results.]

LOCAL FIFTH FORCE MODELS

The Chameleon model represents a fifth force model on a local mass scale. However, in order to evaluate a local mass's sum of the local fifth force coefficient $\equiv \theta_m = \sum\theta_{L_m}$ in terms of the Chameleon model, especially in an atmosphere, a variance was made to the Chameleon model by noting that for objects in the earth's atmosphere, it is more appropriate to modify the thin-shell equation (36) as

$$\Delta R_m \approx 1/3\left(M_E^2/R_m\hat{\beta}_C\rho_m\right)\left(2M_{PL}^4/\rho_0\right)^{1/3}\left(1-\left(\rho_0/\rho_N\right)^{1/3}\right), \tag{37}$$

where the subscript N indicates the Newtonian mass of density ρ_N surrounded by an ambient field of density ρ_0 in which the smaller mass of density ρ_m is also surrounded. Equation (37) is nearly identical to equation (36) since for most objects their densities $\rho_m >> \rho_0 \equiv \rho_{atm}$. This modification is done to allow for changes to an object's thin-shell thickness when placed in an atmosphere near a Newtonian mass of density ρ_N producing the dominate Newtonian force F_N and provides for the redefining of the ratio

$$\Delta R_m \big/ \bar{R}_m \equiv \beta_N^2 \sqrt{l_p \big/ \bar{R}_m} \ , \tag{38}$$

where $\bar{R}_m$ denotes the radial factor of an object and β_N is the local environmental coupling factor between the ambient field of density ρ_0 and the local Newtonian force producing mass of density ρ_N. In the general case, the radial factor $\bar{R}_m$ is the radius of an object; differences will be discussed as required.

Rearranging equation (38) gives the thin-shell thickness for any object as

$$\Delta R_m \approx \beta_N^2 \sqrt{l_p \bar{R}_m} \ , \tag{39}$$

which relates the thin-shell thickness ΔR_m to the product of the square of the local environmental coupling factor β_N and the average between an objects radial factor $\bar{R}_m$ and the Planck length, assuring that $\Delta R_m \big/ \bar{R}_m \to 1$ for insignificantly small objects as noted by Khoury and Weltman (2004a and 2004b) even for large environmental coupling factor β_N.

Using equations (30), (36) and (39), the environmental coupling factor β_N is given by

$$\beta_N \approx \left(1/3\left(M_E^2 \big/ \hat{\beta}_C \rho_N R_N \sqrt{l_p \bar{R}_N}\right)\left(2M_{PL}^4 \big/ \rho_0\right)^{1/3}\left(1-\left(\rho_0/\rho_N\right)^{1/3}\right)\right)^{1/2} . \tag{40}$$

However, for the static ambient field portrayed here, this equation only makes sense if $\hat{\beta}_C \approx 1$, which indicates a 1-to-1 coupling between the Newtonian object's Chameleon mass and its surrounding thin-shell, which then infers a 1-to-1 coupling between ambient field's Chameleon mass and the thin-shell of the Newtonian object. That is, the density of the matter in the Newtonian mass's thin-shell is identical to the density of the Chameleon mass of the surrounding atmosphere. (Thus making searches on the earth impossible.)

Example 3 - Letting $\hat{\beta}_{C_\oplus} \approx 1$, then for the earth with a density $\rho_{N_\oplus} \approx 5520\ kg/m^3$, $\bar{R}_\oplus = 6.37\times 10^6\ m$ and with an atmosphere density $\rho_{0_{atm}} \approx 1.2\ kg/m^3$, equation (40) yields the earth to atmosphere coupling factor $\beta_{N_\oplus} \approx 0.98$. However, letting $\hat{\beta}_C \approx 1$ for the atmosphere with $\bar{R}_{atm} = 6.39\times 10^6\ m$ in our galaxy with a density $\rho_{0_G} \approx 1.45\times 10^{-21}\ kg/m^3$, yields an atmosphere to galactic coupling factor $\beta_{N_{atm}} \approx 2.23\times 10^5$. This indicates that the thin-shells of objects couple greater to the galactic or space Chameleon scalar field than they do to the earth's atmosphere.

Example 4 - For the earth with $\beta_{N_\oplus} \approx 0.98$, equation (39) gives $\Delta R_\oplus \approx 0.97\times 10^{-14}$ for an earth radius $R_\oplus \approx 6.37\times 10^6\ m$ compared to $\Delta R_\oplus \approx 1.04\times 10^{-14}\ m$ given by equation (35) with $\hat{\beta}_{C_\oplus} \approx 1$ and $\phi_{C_{0_{atm}}} \approx 8.87\times 10^{-38}\ kg$. For comparison, equation (39) gives $\Delta R_m \approx 3.85\times 10^{-18}\ m$ for an object of radius $\bar{R}_m \approx 1\ m$ near the earth surface.

Static Ambient Field-Newtonian Mass Model

For a small mass m placed near the surface of a larger mass $M \sim m$ at a distance r_x, where the subscript $\left(_x\right)$ indicates that the distance is from the center of the object, as shown in Figure 4, the field-force $F\left(r_x\right)$ on the smaller object m is given by

$$\begin{aligned} \vec{F}\left(r_x\right) &= mc^2\left(\beta_m/M_{Pl}\right)\partial\left(\nabla\phi\left(r_x\right)\hat{r}\right) \approx 2\beta_m\ \left(3\partial\Delta R_m \big/ \bar{R}_m\right) m\vec{g}_N \\ &= 6\beta_m\left(\partial\Delta R_m \big/ \bar{R}_m\right)\vec{F}_N = \theta_m \vec{F}_N \end{aligned} \ , \tag{41}$$

where $\vec{g}_N \equiv \left(-G_N M/r_x^2\right)\hat{r}$ is the acceleration of gravity due to the large mass with Newtonian force $\vec{F}_N \equiv m\vec{g}_N$ and the change in the gradient $\nabla\phi\left(r_x\right)$ of the ambient (atmosphere) Chameleon field is given from equation (29) as

$$\begin{aligned}\partial\left(\nabla\phi\left(r_x\right)\hat{r}\right) &\approx -3\left(1/4\pi M_{Pl}\right)\left(\left(\Delta R_m+\partial\Delta R_m\right)/\bar{R}_m-\Delta R_m/\bar{R}_m\right)\left(M/r_x^2\right)\left(\hbar/c\right)\hat{r}, \\ &= 2M_{Pl}\left(3\partial\Delta R_m/\bar{R}_m\right)\left(\vec{g}_N/c^2\right),\end{aligned} \tag{42}$$

where the change in the ambient (atmosphere) Chameleon field is due to perturbations caused by time variances in the thin-shell of the larger mass M , which gives rise to a change in the sum of the local fifth force coefficients about the small mass m .

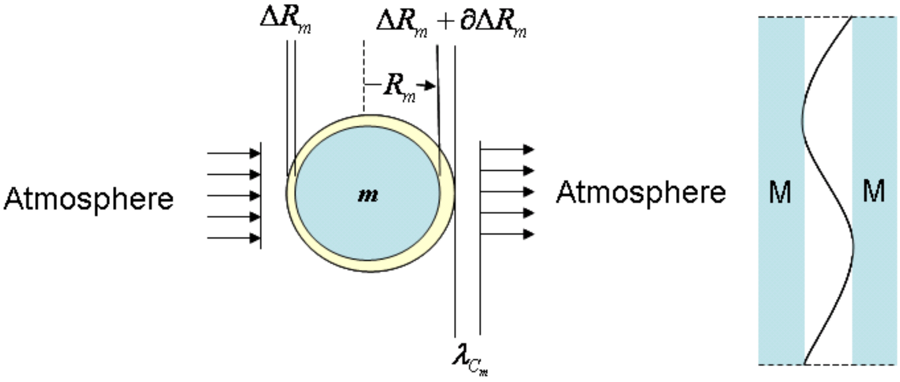

FIGURE 4. Chameleon force model

The change in the sum of local fifth force coefficient is given in like to equation (24) as

$$\begin{aligned}\theta_m &\equiv 1/2\left(+\partial\theta_{L_m}\right)_{right}+1/2\left(-\partial\theta_{L_m}\right)_{left} \\ &\approx 6\beta_m\left(\left(\Delta R_m+\left(\partial\Delta R_m\right)_{right}\right)/\bar{R}_m-\left(\Delta R_m-\left(\partial\Delta R_m\right)_{left}\right)/\bar{R}_m\right)=6\beta_m\left(\partial\Delta R_m/\bar{R}_m\right),\end{aligned} \tag{43}$$

using equation (41) and given that $\left(\partial\Delta R_m\right)_{left}=0$. That is, unlike the fifth force model of Figure 2 from which equation (24) is derived, the sum of the fifth force coefficients in the Chameleon model are taken with respect to the change of the thin-shell thicknesses according to $\text{Decreases}\equiv-\partial\theta_{L_m}=-6\beta_m\left(\left(\Delta R_m-\partial\Delta R_m\right)/\Delta R_m\right)$ or $\text{Increases}\equiv+\partial\theta_{L_m}=6\beta_m\left(\left(\Delta R_m+\partial\Delta R_m\right)/\Delta R_m\right)$; implying an increase $\left(-\partial\theta_L\right)$ or decrease $\left(+\partial\theta_L\right)$ in the Chameleon field-force on an object's thin-shell.

Equation (41) also gives

$$\partial\Delta R_m \approx 1/6\left(1/\beta_m\right)\left|\vec{F}\left(r_x\right)/\vec{F}_N\right|\bar{R}_m, \tag{44}$$

which for a static conditions, where the object is stationary relative to both the ambient (atmosphere) Chameleon field and the larger mass, gives a fixed radial factor and field-force $F\left(r_x\right)$. However in Figure 4, it must be assumed that there is a change in the ambient Chameleon field, which indicates a changing Chameleon factor $\hat{\beta}_C$. That is, the field to mass coupling factor β_m changes across the smaller mass, where this change is given in similar form to equation (40) as

$$\partial\beta_m \approx \left(1/3\left(M_E^2/\partial\hat{\beta}_C\rho_m\bar{R}_m\sqrt{l_p\bar{R}_m}\right)\left(2M_{PL}^4/\rho_0\right)^{1/3}\left(1-\left(\rho_0/\rho_m\right)^{1/3}\right)\right)^{1/2} \tag{45}$$

and also implies a change to the Newtonian coupling factor as

$$\partial\beta_N \approx \left(1/3\left(M_E^2/\partial\hat{\beta}_C\rho_N\bar{R}_N\sqrt{l_p\bar{R}_N}\right)\left(2M_{PL}^4/\rho_0\right)^{1/3}\left(1-\left(\rho_0/\rho_N\right)^{1/3}\right)\right)^{1/2}, \tag{46}$$

which allows the radial change to be given from equation (39) as

$$\partial\Delta R_m \approx \partial\beta_N^2\sqrt{l_p\bar{R}_m} \tag{47}$$

and the field-force $\vec{F}_m$ on the mass m to be given by equation (41) as

$$\vec{F}_m \equiv \vec{F}(r_x) = \theta_m \vec{F}_N \approx \left(6\partial\beta_m \partial\beta_N^2 \sqrt{l_p / \overline{R}_m}\right)\vec{F}_N \tag{48}$$

with respect to the sum of the local fifth force coefficient of equation (43).

Example 5 - For the earth, the field-force $\vec{F}_m$ is extremely small where we can let $\partial\hat{\beta}_C \approx 1$, then for $\rho_{0_{atm}} \approx 1.2\ kg/m^3$ and $\vec{g}_N \approx 9.82\ m/s^2$, which gives $\partial\beta_{N_\oplus} \approx 1$. Then for objects with a radial factor $\overline{R}_m \approx 1\,m$, gives $\vec{F}_m \approx 6\partial\beta_m \sqrt{l_p} m\vec{g}_N$ from equation (48) with $\vec{F}_N = m\vec{g}_N$. Then for the following objects:

Wood, with a density $\rho_{wood} \approx 500\ kg/m^3$, $m_{wood} \approx 2{,}094\ kg$, which equation (45) gives $\beta_{wood} \approx 3.96\times10^5$ and $\vec{F}_{wood} \approx 1.96\times10^{-7}\ N$;
Object with a density $\rho_m \approx 5520\ kg/m^3$, $m \approx 23{,}122\ kg$, which equation (45) gives $\beta_m \approx 1.24\times10^5$ and $\vec{F}_m \approx 6.80\times10^{-7}\ N$;
Gold with a density $\rho_{gold} = 19330\ kg/m^3$, $m_{gold} \approx 80{,}969\ kg$, which equation (45) gives $\beta_{gold} \approx 6.70\times10^4$ and $\vec{F}_{gold} \approx 1.29\times10^{-6}\ N$.

The field-force difference between wood and gold is only approximately a factor of 10 for the same radial factor (*i.e.*, size & shape) and shows that the mass of the object adds very little to the field-force as was noted by Khoury and Weltman in their Chameleon model.

Non Static Ambient Field-Static Newtonian Mass Model: The Chameleon Universe Expansion Model

In the previous section it was shown that the thin-shell mechanism implies a change in the thin-shell thickness about a stationary object of mass m due to the presence of a stationary and dominate Newtonian force provider of mass $M \gg m$, where both masses are in a static ambient (atmosphere) Chameleon field of density ρ_0. In this section, the effect of a flowing Chameleon field on a static object under the influence of a static dominate Newtonian force provider is considered. This happens to be the case of cosmological expansion, where the flowing ambient Chameleon field is represented by the universe vacuum energy density ρ_v given by equation (3) and the Newtonian force provided is represented by the universe matter density ρ_M given by equation (2).

That is, the galaxy of Figure 2 is pushed by the change in the flowing vacuum energy ρ_v across the galaxy to produce an expansion force F_{Exp} on the galaxy away from the baryonic center of the universe, where the pressure is expanding - moving faster than the galaxy. Then by placing the galaxy far from any other mass, where the Newtonian gravity influence can be neglected, there would be the normal thin-shell thickness ΔR_G on one side and a change $\partial\Delta R_G$ of the thin-shell thickness on the other side to compensate for the change in the vacuum energy as shown in Figure 5, where the dotted line represents the initial static state of the thin-shell thickness.

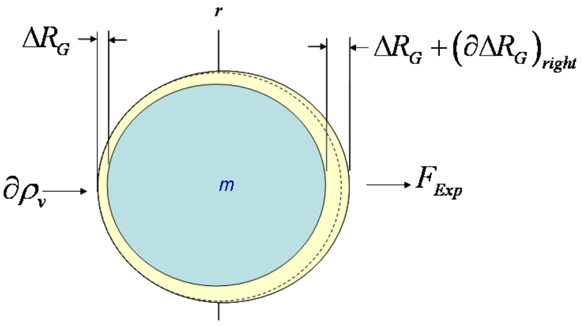

FIGURE 5. Expansion Model

Then on the left side of the galaxy in Figure 5 the Chameleon field-force $F_{left} > F_{right}$, where the left side fifth force coefficient $\left(-\theta_{L_G}\right)_{left}$ is given as

$$\left(-\theta_{L_G}\right)_{left} \approx -6\partial\beta_{m_G} \tag{49}$$

and on the right side the Chameleon field-force $F_{right} < F_{left}$, where the right side fifth force coefficient $\left(+\theta_{Exp}\right)_{right}$ is given as

$$\left(+\theta_{L_G}\right)_{right} \approx 6\partial\beta_{m_G}\left(1+\partial\Delta R_G/\bar{R}_G\right). \tag{50}$$

Then using equation (43) and (47), equations (49) and (50) give the sum of the fifth force coefficients about the galaxy as

$$\theta_G \approx 6\partial\beta_{m_G}\left(\partial\Delta R_G/\bar{R}_G\right) = 6\partial\beta_{m_G}\partial\beta_N^2\sqrt{l_p/\bar{R}_G}\ , \tag{51}$$

which gives the expansion force in like to equation (48) as

$$\vec{F}_v = -\theta_G\vec{F}_N \approx -6\partial\beta_{m_G}\partial\beta_N^2\sqrt{l_p/\bar{R}_{m_G}}\ \vec{F}_N\ , \tag{52}$$

where $\vec{F}_N$ is given for the universe by equation (13) and where the negative sign is directional as the change to the thin-shell is away from the baryonic center of the universe. Now from equation (26), equation (52) implies that the sum of the fifth force coefficients about the galaxy

$$\theta_G \approx 6\partial\beta_{m_G}\partial\beta_{N_G}^2\sqrt{l_p/\bar{R}_{m_G}} \approx 2\left(R/g_{m_G}\bar{R}_{m_G}^2\right)\left(\rho_v/\rho_M\right) \equiv 2\left(\rho_{v_G}/\rho_M\right), \tag{53}$$

where $\rho_{v_G} \equiv \rho_v R/g_{m_G}\bar{R}_{m_G}^2$ and which further implies that

$$\rho_{v_G}/\rho_M \approx 3\partial\beta_{m_G}\partial\beta_{N_G}^2\sqrt{l_p/\bar{R}_{m_G}} \tag{54}$$

or

$$\partial\beta_{m_G}\partial\beta_{N_G}^2 \approx 1/3\left(\rho_{v_G}/\rho_M\right)\sqrt{\bar{R}_{m_G}/l_p}\ . \tag{55}$$

The coupling factors β_{m_G} and β_{N_G} are defined from equation (40) as

$$\begin{aligned}\partial\beta_{m_G} &\approx \left(1/3\left(M_E^2/\partial\hat{\beta}_C\rho_G\bar{R}_G\sqrt{l_p\bar{R}_G}\right)\left(2M_{PL}^4/\rho_{v_G}\right)^{1/3}\left(1-\left(\rho_{v_G}/\rho_G\right)^{1/3}\right)\right)^{1/2} \\ &= \left(1/3\left(M_E^2/\partial\hat{\beta}_C\rho_G\bar{R}_G\sqrt{l_p\bar{R}_G}\right)\left(2M_{PL}^4/\rho_{v_G}\right)^{1/3}\alpha_{m_G}\right)^{1/2}\end{aligned} \tag{56}$$

and

$$\begin{aligned}\partial\beta_{N_G} &\approx \left(1/3\left(M_E^2/\partial\hat{\beta}_C\rho_c\bar{R}_U\sqrt{l_p\bar{R}_U}\right)\left(2M_{PL}^4/\rho_{v_G}\right)^{1/3}\left(1-\left(\rho_{v_G}/\rho_c\right)^{1/3}\right)\right)^{1/2} \\ &= \left(1/3\left(M_E^2/\partial\hat{\beta}_C\rho_c\bar{R}_U\sqrt{l_p\bar{R}_U}\right)\left(2M_{PL}^4/\rho_{v_G}\right)^{1/3}\alpha_{N_G}\right)^{1/2}\end{aligned} \tag{57}$$

where ρ_c is the total energy density of the Universe and ρ_{v_G} is the vacuum density about the galaxy and

$$\alpha_{N_G} \approx 1-\left(\rho_{v_G}/\rho_c\right)^{1/3}; \quad \alpha_{m_G} \approx 1-\left(\rho_{v_G}/\rho_G\right)^{1/3} \tag{58}$$

Example 6 – Given $\rho_c \approx 10^{-26}\ kg/m^3$, $\rho_{v_G} \approx 5.14\times10^{-31}\ kg/m^3$ and $\rho_G \approx 1.45\times10^{-21}\ kg/m^3$, equation (58) gives $\alpha_{N_G} \approx 0.9628$ and $\alpha_{m_G} \approx 0.9993$.

Now by combining equation (53) with equations (56) and (57) yields for convenience, the factor

$$\theta'_G \equiv \theta_G\sqrt{\partial\hat{\beta}_C^3\ \alpha_{N_G}^{-2}\ \alpha_{m_G}} \approx 2M_E^3M_{PL}^2\left(\rho_c\bar{R}_U\sqrt{\bar{R}_U\bar{R}_{m_G}}\right)^{-1}\left(2/3\left(\rho_G\bar{R}_G\sqrt{l_p\bar{R}_G}\right)^{-1}\rho_{v_G}^{-1}\right)^{1/2}. \tag{59}$$

Combining equation (59) back with equation (53) yields

$$\partial\hat{\beta}_C \approx \left(\alpha_{N_G}^2 \alpha_{m_G} \left(\frac{1}{2} \theta_L \left(\rho_M / \rho_{v_G} \right) \right)^2 \right)^{1/3}, \tag{60}$$

which is shown in the following example to $\neq 1$; indicating that for an object in motion $\partial\hat{\beta}_C \neq 1$; whereby $\partial\hat{\beta}_C$ is redefined hereafter as the motion coupling factor.

> **Example 7** – Using the values of **Example 6**, with the galaxy radial factor $\overline{R}_G \approx 3.31 \times 10^{20}\ m$ and $\overline{R}_U \approx 1.03 \times 10^{27}\ m$, equation (59) gives $\theta'_G \approx 5.63 \times 10^{-11}$ and equation (60) gives $\partial\hat{\beta}_C \approx 2.73 \times 10^{-5}$, which then from equation (56) gives $\partial\beta_m \approx 2.24 \times 10^{9}$ and from equation (57) gives $\partial\beta_N \approx 1.13 \times 10^{7}$ as a check using these values give $\theta_G \approx 3.80x10^{-4}$ from equation (59) and (53) as it should for $\rho_M \approx 2.70 \times 10^{-27}\ kg/m^3$.

Changing Ambient Field & Static Newtonian Mass Model: Mass Collision Model

A large mass m_1 and smaller mass m_2 are placed in a static ambient field of density ρ_0 with a local environmental coupling factor β_N under a local Newtonian force $\vec{F}_N$ non-related to either mass. The larger mass m_1 has an acceleration a_1 producing a force $\vec{F}_1 = m_1\vec{a}_1$ and approaching the stationary smaller mass m_2. This is illustrated in Figure 6 for a thin cross-sectional slice of both masses just before the smaller mass m_2 begins to move - ignoring deformation - due to the collision, under a force $\vec{F}_2 = m_2\vec{a}_2$.

As shown, the approach of the larger mass m_1 toward the smaller mass m_2 causes a transfer of the energy - denoted by $\partial\Delta R'_1$ - from the moving mass's increased thin-shell $\Delta R_1 + \partial\Delta R_1$, which caused its motion, to the smaller mass's thin-shell. However, the large mass's thin-shell between the two masses prevents the smaller mass's thin-shell from increasing, whereby the energy is transferred to the opposite side. Each mass in this model then exhibit the same characteristics as in the gravitational attraction model of Figure 4.

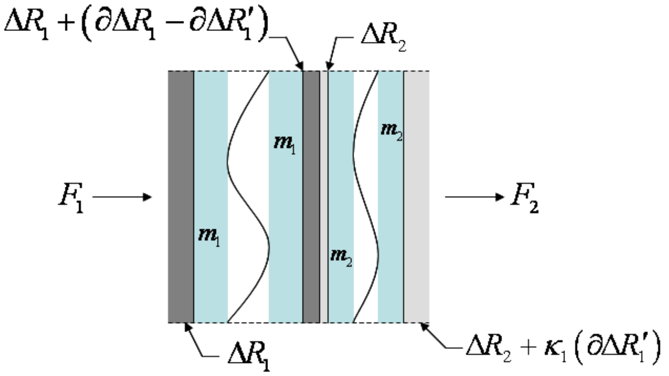

FIGURE 6. Mass Collision Model – Thin slice of a larger mass m_1 colliding with a smaller mass m_2

In this model, the larger mass will continue to move toward the smaller mass until $\partial\Delta R_1 - \partial\Delta R'_1 = 0$ and the smaller mass will not move until $\Delta R_2 + \kappa_1(\partial\Delta R'_2) > \Delta R_1$ where κ_1 is a geometric factor due the radial difference between the to masses.

Collision Field-Force Analysis

Determining the field-force on either the small or large mass after the collision can be accomplished by viewing either mass as being in motion relative to the other. Whereby, the calculation of the field force of either mass will have similar form, but will have different coupling factor values related to how a mass views the motion of other. For example, the field-force $\vec{F}_2$ induced on the smaller mass m_2 by the collision with the larger mass m_1 is given in like to the attractive mass case of equation (48) as

$$\vec{F}_2 = \theta_2 \vec{F}_N = 6\partial\beta_2 \partial\beta_{N_2}^2 \sqrt{l_p / \overline{R}_2}\ \vec{F}_N \tag{61}$$

and the field-force $\vec{F}_1'$ induced on the larger mass m_1 by the collision with the smaller mass m_2 is given

$$\vec{F}_1' = -\theta_1 \vec{F}_N = -6\partial\beta_1 \partial\beta_{N_1}^2 \sqrt{l_p / \overline{R}_1}\ \vec{F}_N\,. \tag{62}$$

In the following analysis only the field-force on the stationary smaller mass is considered, where the coupling factors $\partial\beta_2$ and $\partial\beta_{N_2}$ are given with respect to equation (40) as

$$\partial\beta_2 \approx \left(1/3\left(M_E^2 \big/ \partial\hat{\beta}_C \rho_2 \overline{R}_2 \sqrt{l_p \overline{R}_2}\right)\left(2M_{PL}^4/\partial\rho_0\right)^{1/3}\left(1-\left(\partial\rho_0/\rho_2\right)^{1/3}\right)\right)^{1/2}, \tag{63}$$

and

$$\partial\beta_{N_2} \approx \left(1/3\left(M_E^2 \big/ \partial\hat{\beta}_C \partial\rho_{2_L} \overline{R}_{2_L} \sqrt{l_p \overline{R}_{2_L}}\right)\left(\left(2M_{PL}^4/\partial\rho_0\right)^{1/3}\left(1-\left(\partial\rho_0/\partial\rho_{2_L}\right)^{1/3}\right)\right)\right)^{1/2}, \tag{64}$$

Since the force $\vec{F}_2$ on the smaller object m_2 will be dominated by the applied force $\vec{F}_1$, the change in the local density $\partial\rho_{2_L}$ approaches that of the moving larger mass m_2 and is defined as

$$\partial\rho_{2_L} \to \partial\rho_1 \approx \left|\vec{F}_1 / \vec{F}_N\right| \rho_1 = \left|\vec{a}_1 / \vec{g}_N\right| \rho_1 = 3m_1 / 4\pi\partial\overline{R}_1^3\,, \tag{65}$$

where $\partial\overline{R}_1$ implies a change to the large mass's radial factor $\overline{R}_1$ and is given from equation (65) as

$$\partial\overline{R}_1 \approx \left(\left|\vec{g}_N / \vec{a}_1\right|\right)^{1/3} \overline{R}_1\,. \tag{66}$$

Equation (64) is then rewritten as

$$\partial\beta_{N_2} \approx \left(1/3\left(M_E^2 \big/ \partial\hat{\beta}_C \partial\rho_1 \partial\overline{R}_1 \sqrt{l_p \partial\overline{R}_1}\right)\left(2M_{PL}^4/\partial\rho_0\right)^{1/3}\left(1-\left(\partial\rho_0/\partial\rho_1\right)^{1/3}\right)\right)^{1/2}. \tag{67}$$

The change $\partial\overline{R}_1$ in the large mass's radial factor $\overline{R}_1$ implies that the ambient density change $\partial\rho_0$ approaches that of the moving larger mass m_1, whereby $\partial\rho_0 \to (\partial\rho_1 - \rho_0)$, where equation (67) can be reduced to

$$\begin{aligned}
\partial\beta_{N_2} &\approx \left(1/3\left(M_E^2 \big/ \partial\hat{\beta}_C \partial\rho_1 \partial\overline{R}_1 \sqrt{l_p \partial\overline{R}_1}\right)\left(2M_{PL}^4/\left(\partial\rho_1 - \rho_0\right)\right)^{1/3}\left(1-\left(\left(\partial\rho_1 - \rho_0\right)/\partial\rho_1\right)^{1/3}\right)\right)^{1/2} \\
&\approx \left(1/3\left(M_E^2 \big/ \partial\hat{\beta}_C \partial\rho_1 \partial\overline{R}_1 \sqrt{l_p \partial\overline{R}_1}\right)\left(2M_{PL}^4/\left(\partial\rho_1 - \rho_0\right)\right)^{1/3}\left(1-\left(1 - \rho_0/\partial\rho_1\right)^{1/3}\right)\right)^{1/2}, \\
&\approx \left(1/3\left(M_E^2 \big/ \partial\hat{\beta}_C \partial\rho_1 \partial\overline{R}_1 \sqrt{l_p \partial\overline{R}_1}\right)\left(2M_{PL}^4/\left(\partial\rho_1 - \rho_0\right)\right)^{1/3}\alpha_1\right)^{1/2}
\end{aligned} \tag{68}$$

and equation (63) can be given as

$$\begin{aligned}
\partial\beta_2 &\approx \left(1/3\left(M_E^2 \big/ \partial\hat{\beta}_C \rho_2 \overline{R}_2 \sqrt{l_p \overline{R}_2}\right)\left(2M_{PL}^4/\left(\partial\rho_1 - \rho_0\right)\right)^{1/3}\left(1-\left(1-\left(\partial\rho_1 - \rho_0\right)/\rho_2\right)^{1/3}\right)\right)^{1/2} \\
&\approx \left(1/3\left(M_E^2 \big/ \partial\hat{\beta}_C \rho_2 \overline{R}_2 \sqrt{l_p \overline{R}_2}\right)\left(2M_{PL}^4/\left(\partial\rho_1 - \rho_0\right)\right)^{1/3}\alpha_2\right)^{1/2}
\end{aligned}, \tag{69}$$

where

$$\alpha_1 = 1-\left(1-\rho_0/\partial\rho_1\right)^{1/3}; \quad \alpha_2 = 1-\left(1-\left(\partial\rho_1 - \rho_0\right)/\rho_2\right)^{1/3}. \tag{70}$$

Example 8 – Given objects with density $\rho_1 = \rho_2 \approx 5520\ kg/m^3$ and $\overline{R}_1 = \overline{R}_2 \approx 0.076\ m$ in the earth's atmosphere of density $\rho_{0_{atm}} \approx 1.2\ kg/m^3$ and $\vec{g}_N \approx 9.82\ m/s^2$, where one of the objects is stationary, say the object with mass m_2, while the other, say the object with mass m_1, is moving toward the mass m_2, with acceleration $a_1 \approx 20\ m/s^2$; equation (65) gives $\partial\rho_1 \approx 11{,}200\ kg/m^3$, equation (66) gives $\partial\overline{R}_1 \approx 0.0597\ m$ and equation (70) gives $\alpha_1 \approx 3.56\times10^{-5}$ and $\alpha_2 \approx 2.01$, which are both positive.

Combining equations (68) and (69) with equation (61) yields the sum of the fifth force coefficients as

$$\theta_2 \approx 2\sqrt{(\alpha_1)^2 \alpha_2 \Big/ \left(\partial\hat{\beta}_C\right)^3} \left(M_E^3 \Big/ \partial\rho_1 \partial\bar{R}_1 \sqrt{\partial\bar{R}_1 \bar{R}_2}\right) \left(1/3\left(\rho_2 \bar{R}_2 \sqrt{l_p \bar{R}_2}\right)^{-1} \left(2M_{PL}^4 \big/ (\partial\rho_1 - \rho_0)\right)\right)^{1/2} \tag{71}$$

or the factor

$$\theta_2' = \theta_2 \sqrt{\left(\partial\hat{\beta}_C\right)^3 \Big/ (\alpha_1)^2 \alpha_2} \approx 2\left(M_E^3 M_{PL}^2 \Big/ \partial\rho_1 \partial\bar{R}_1 \sqrt{\partial\bar{R}_1 \bar{R}_2}\right) \left(2/3\left(\rho_2 \bar{R}_2 \sqrt{l_p \bar{R}_2}\right)^{-1} \left(\partial\rho_1 - \rho_0\right)^{-1}\right)^{1/2} . \tag{72}$$

Combining equation (72) back with equation (61) yields the motion coupling factor

$$\partial\hat{\beta}_C = \left((\alpha_1)^2 \alpha_2 \left(\theta_L \left|\vec{F}_N / \vec{F}_2\right|\right)^2\right)^{1/3} \tag{73}$$

Example 9 - An object of mass $m_1 = 10\ kg$ and initial velocity $v_{1\,\text{int}} = 10\ m/s$ hits an identical stationary object m_2 with initial $v_{2\,\text{int}} = 0\ m/s$. Now considering a perfectly elastic “head-on” collision between the two objects, the objects separate after collision and have different final velocities, v_{1f} and v_{2f}. Since the kinetic energy and momentum are both conserved, we have

Conservation of kinetic energy: $\tfrac{1}{2} m_1 v_{1\,\text{int}}^2 + \tfrac{1}{2} m_2 v_{2\,\text{int}}^2 \cdot \tfrac{1}{2} m_1 v_{1\,\text{int}}^2 = \tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2$
Conservation of momentum: $m_1 v_{1\,\text{int}} + m_2 v_{2\,\text{int}} = m_1 v_{1f} + m_2 v_{2f}$

These equations give:

$$v_{1_f} = \left(2m_2 v_{2\,\text{int}} + v_{1\,\text{int}}(m_1 - m_2)\right) \big/ (m_1 + m_2) = \left(2*10*0 + 10(10-10)\right) \big/ (10+10) = 0\ m/s$$

and

$$v_{2_f} = \left(2m_1 v_{1\,\text{int}} - v_{2\,\text{int}}'(m_1 - m_2)\right) \big/ (m_1 + m_2) = \left(2*10*10 - 0*(10-10)\right) \big/ (10+10) = 10\,m/s\ ;$$

indicating that the first mass m_1 comes to rest while the second mass m_2 moves off with a velocity equal to the original velocity of the mass m_1. That is, $F_2 = F_1$ and $F_1' = 0$.

Now using the values of **Example 9** and given that the force of the collision $F_1 \approx 200\ N$, where at the time of collision, the acceleration $a_1 = F_1/m_1 \approx 20\ m/s^2$. Then equation (72) gives $\theta_2' \approx 0.444$, equation (73) with $F_L = F_2$ gives $\partial\hat{\beta}_C \approx 4.95\times10^{-4}$, equation (68) gives $\partial\beta_{N_2} \approx 4.34\times10^4$ and equation (69) gives $\partial\beta_2 \approx 1.23\times10^7$ for this example, as a check using equation (71) gives $\theta_2 \approx 2.04$ and equation (61) gives $F_2 \approx 200\ N$ as it should.

The author wishes to note that this example only holds for the case of two identical masses. More analysis on the effect of dissimilar mass collision is needed to understand the effects on these coupling factors. This example is presented as it is needed for the section on rocket propulsion.

CONCLUSION

This paper investigated the notion that our universe is composed of scalar fields and that such notion can be used to calculate the same forces as attainable from the standard Newtonian force models. This was done through the development of fifth force models base on the Chameleon scalar field model present by Khoury and Weltman (2004a and 2004b) with further work by Brax *et al* (2004a and 2004b).

The author notes that these models are incomplete, but with further development, could lead others to develop force producing devices using unforeseen methods not visible under our current models.

NOMENCLATURE

α = grouping factor
a = acceleration (m/s^2)
Λ = cosmological constant (m^{-2})
β = coupling factor
$\hat{\beta}_C$ = Chameleon (thin-shell) coupling factor
β_m = Newtonian (mass to field) coupling factor
β_N = field to mass coupling factor
ρ_m = local mass density (kg/m^3)
ρ_0 = ambient mass density (kg/m^3)
ρ_M = total matter density (kg/m^3)
ρ_v = vacuum energy density (J/m^3)
ρ_c = total matter plus energy density (kg/m^3)
E_p^M = matter potential energy (J)
E_p^v = vacuum potential energy (J)
E_{tot} = total vacuum energy (J)
F = force (N)
F_N = Newtonian gravitational force [$\equiv mg \;\; (N)$]
F_v = vacuum expansion force (N)
H = Hubble parameter (s^{-2})
D = relative distance (m)
k = topological constant (-1, 0, 1)

κ = geometric factor
m = small mass (kg)
m_{C_m} = internal Chameleon field mass (m)
M = large mass (kg)
M_E = Cosmological Energy Factor (m^{-1})
p = vacuum pressure (N/m^2)
Φ_N = Newtonian (gravitational) potential
r = radius (m)
R = mass radius (m)
R_0 = Universe radius (m)
$\bar{R}$ = radial factor (m)
ΔR = Thin shell thickness (m)
$\partial\Delta R$ = small change to the thin shell thickness (m)
$\dot{R}$ = cosmological velocity (m/s)
$\ddot{R}$ = universe acceleration factor (m/s^2)
θ_m = sum of the local fifth force coefficients
θ_{L_i} = local fifth force coefficient
t = time (s)
v = velocity (m/s)
ϕ_C = internal Chameleon scalar field (kg)
λ_C = Compton wave length or effective range

θ_{Exp} = sum of the universe expansion fifth force coefficients
$V_\phi(r,t)$, V_ϕ = scalar field energy potential density (J/m^3)
$x(r)$, x = distance (m) as function of radius r
$\phi(r,t), \phi$ = scalar field

Constants

c = speed of light [$\approx 2.99\times10^{8} \;\; m/s$]
M_E = energy scale [$\approx 10^{4} \;\; m^{-1}$]
M_{PL} = Reduced Planck mass [$\approx 4.34\times10^{-9} \;\; kg$]

G_N = gravitational constant [$\approx 6.67\times10^{-11}\, m^3/kg\cdot s^2$]
$\hbar$ = Planck constant [$\approx 6.626068\times10^{-34}\, m^2\cdot kg/s$]

ρ_c = critical density [$\approx 10^{-26} \;\; kg/m^3$]
g = Earth's gravity [$= 9.8 \;\; m/s^2$]
l_p = Planck length [$\approx 1.616252\times10^{-35} \;\; m$]

Subscripts

atm = atmosphere
i = given mass (0, m)
int = initial
m = local mass
0 = Ambient field
1, 2 = given interacting mass

N = Newtonian
G = galaxy
Exp = expansion
v = vacuum
$\oplus$ = Earth

∞ = infinity
U = Universe
C = Chameleon
c = Chameleon mass

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