The Chameleon Solid Rocket Propulsion Model

Glen A. Robertson

Institute for Advanced Studies in the Space, Propulsion & Energy Sciences
265 Ita Ann
Madison, AL 35757
256-694-7941; gar@ias-spes.org

Abstract. The Khoury and Weltman (2004a and 2004b) Chameleon Model presents an addition to the gravitation force and was shown by the author (Robertson, 2009a and 2009b) to present a new means by which one can view other forces in the Universe. The Chameleon Model is basically a density-dependent model and while the idea is not new, this model is novel in that densities in the Universe to include the vacuum of space are viewed as scalar fields. Such an analogy gives the Chameleon scalar field, dark energy/dark matter like characteristics; fitting well within cosmological expansion theories. In respect to this forum, in this paper, it is shown how the Chameleon Model can be used to derive the thrust of a solid rocket motor. This presents a first step toward the development of new propulsion models using density variations verse mass ejection as the mechanism for thrust. Further, through the Chameleon Model connection, these new propulsion models can be tied to dark energy/dark matter toward new space propulsion systems utilizing the vacuum scalar field in a way understandable by engineers, the key toward the development of such systems. This paper provides corrections to the Chameleon rocket model in Robertson (2009b).

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INTRODUCTION

Before the engineering community can actively begin the development of new propulsion systems outside current models, new force mechanisms are needed. Such mechanisms must inherently cross several scientific disciplines, but be founded in the current Newtonian physics and extend into the Einstein and Quantum Field Theories. One such mechanism that has achieves this, is the Khoury and Weltman (2004a and 2004b) Chameleon Model. According to Khoury and Weltman, the Chameleon Model presents a small force addition to gravity on earth, but has large consequences on the galactic scale, which infers a strong dark matter/energy connection. The author extended this to other Newtonian forces through the definition of coupling factors, which were set to a value of ≈ 1 in the Khoury and Weltman analysis. The author (Robertson, 2009a and 2009b) showed that these coupling factors change dependent on how an object's or the external environment's density changes with motion. Whereby, coupling factors ≈ 1 are acceptable in the Khoury and Weltman analysis as static objects and environments were the main focus.

In the following, the Chameleon Model is reviewed to include the development of coupling factors that define specific characteristics of a force system with respect to local and environmental densities. Second a modified Chameleon Model is discussed. This is followed by the application of the Modified Chameleon Model to solid rocket propulsion.

THE CHAMELEON MODEL

The Chameleon Model is basically a density-dependent model and while the idea is not new (see for example: Hill and Ross, 1988; Ellis *et al*, 1989; Wetterich, 1995; Anderson and Carrol, 1997; Damour and Polyakov, 1994a and 1994b; Huey *et al*, 2000), this model is novel in that densities in the Universe to include the vacuum of space are viewed as scalar fields, referred to as the Chameleon field. Such an analogy gives the Chameleon field dark energy/dark matter like characteristics (Brax *et. al*., 2007); fitting well within the cosmological expansion (see: Robertson, 2009a).

Dark Matter in the Chameleon Model

The dark energy connection to the Chameleon Model is seen by using the internal Chameleon Field Mass

$$m_{C_m} = \left(\frac{\sqrt{6}}{M_E}\right)\left(\frac{\beta_{C_m}\cdot\rho_m}{2M_{Pl}}\right)^{2/3}\left(\frac{\hbar}{c}\right), \tag{1}$$

where $M_E \approx 10^4\ m^{-1}$ is the cosmological energy scale factor, $M_{Pl} \equiv \sqrt{\hbar c/8\pi G_N}$ is the reduced Planck mass, $\rho_m = 3m/4\pi R_m^3$ is the density of an object and β_{C_m} is a coupling factor to be discussed later.

With $\beta_{C_m} \approx 1$, equation (1) then gives results on the order of ultra low atomic matter $(\sim 10^{-3} eV)$, when the density of the earth is applied and on the order of the dark matter particle size $(\sim 10^{-23} eV)$ indicated by Silverman and Mallett (2001), when the density of the universe is applied.

Thin-Shell Mechanism

The foundation of the Chameleon Model is the thin-shell mechanism, Figure 1, which was developed by concentrating on the static solution where $t = 0$, such that, the external scalar Chameleon field was derived to be

$$\phi(r, t=0) \approx -m\left(\frac{1}{4\pi M_{Pl}}\right)\left(\frac{3\Delta R_m}{R_m}\right)\left(\frac{e^{-r/\lambda_{C_\infty}}}{r}\right)\left(\frac{\hbar}{c}\right) + \phi_{C_0} \tag{2}$$

about a spherically symmetric and significantly large object of homogeneous density ρ_m, mass m and radius R_m having a thin-shell thickness ΔR_m. *The thin-shell thickness $\Delta R_m << R_m$ and is shown large in Figure 1 for clarity.* The subscript $(_\infty)$ in equation (2) is to denote the external Chameleon field far from an object. *The original Khoury and Weltman version of equation (2) contains an additional coupling factor term, which they set to 1. Here this coupling factor is combined within ΔR_m in equation (12).*

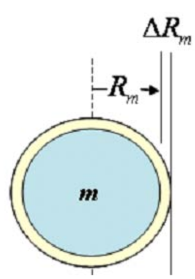

FIGURE 1. Chameleon thin-shell model.

The effective internal Chameleon field of an object is given by

$$\phi_{C_m} = M_E^2 \left(\frac{2M_{PL}}{\beta_{C_m} \rho_m} \right)^{1/3} \left(\frac{\hbar}{c} \right), \tag{3}$$

where the object's thin-shell thickness ΔR_m minimizes the effective field potential $\partial V_\phi(r,t=0) / \partial\phi + \rho_m e^{\phi_c/M_{Pl}} / M_{Pl} = 0$ and holds everywhere inside the object except within a thin shell of thickness ΔR_m at the surface, where the Chameleon field potential rapidly grows to compensate for the higher ambient Chameleon field ϕ_{C_0} about the object. *Equation (3) applies to all matter densities to include the surrounding matter density or ambient field* ϕ_{C_0} *or atmosphere, which could be the space vacuum.*

Thin-Shell Thickness

The thin shell thickness ΔR_m of an object of mass m is related to the surrounding or ambient Chameleon field ϕ_{C_0}, the internal field ϕ_{C_m} and the local Newtonian Gravitational potential

$$\Phi_N = \left(\frac{G_N m}{R_m} \right) c^{-2} = g_N R_m c^{-2} \tag{4}$$

near the object by

$$3\Delta R_m / R_m = \left(\frac{\phi_{C_0} - \phi_{C_m}}{6M_{Pl}} \right) \left(\frac{1}{\Phi_N} \right) = \left(\frac{\phi_{C_0} - \phi_{C_m}}{6M_{Pl}} \right) \left(\frac{c^2}{g_N R_m} \right), \tag{5}$$

where equation (5) yields the object's thin-shell thickness

$$\Delta R_m \approx \left(\frac{\phi_{C_0} - \phi_{C_m}}{18M_{Pl}} \right) \left(\frac{c^2}{g_N} \right) = 1/3 \left(\frac{\phi_{C_0} - \phi_{C_m}}{\rho_m R_m} \right) \left(\frac{c}{\hbar} \right) M_{PL}, \tag{6}$$

or by incorporating equation (3) as

$$\begin{aligned} \Delta R_m &\approx 1/3 \left(\frac{M_E^2}{\rho_m R_m} \right) \left(\left(\frac{2M_{PL}^4}{\beta_{C_0} \rho_0} \right)^{1/3} - \left(\frac{2M_{PL}^4}{\beta_{C_m} \rho_m} \right)^{1/3} \right) \\ &\approx 1/3 \left(\frac{M_E^2}{\rho_m R_m} \right) \left(2M_{PL}^4 \right)^{1/3} \left(\left(\frac{1}{\beta_{C_0} \rho_0} \right)^{1/3} - \left(\frac{1}{\beta_{C_m} \rho_m} \right)^{1/3} \right) \\ &\approx 1/3 \left(\frac{M_E^2}{\rho_m R_m} \right) \left(\frac{2M_{PL}^4}{\rho_0} \right)^{1/3} \left(\left(\frac{1}{\beta_{C_0}} \right)^{1/3} - \left(\left(\frac{\rho_0}{\rho_m} \right) \frac{1}{\beta_{C_m}} \right)^{1/3} \right), \\ &\approx 1/3 \left(\frac{M_E^2}{\hat{\beta}_{C_m} \rho_m R_m} \right) \left(\frac{2M_{PL}^4}{\rho_0} \right)^{1/3} \end{aligned} \tag{7}$$

where

$$\frac{1}{\hat{\beta}_{C_m}} = \left(\frac{1}{\beta_{C_0}} \right)^{1/3} - \left(\left(\frac{\rho_0}{\rho_m} \right) \frac{1}{\beta_{C_m}} \right)^{1/3}.$$

In Robertson (2009a), equations (6) and (7) were shown to produce near equivalent results for the Earth with $\beta_{C_m} \approx \beta_{C_0} \approx 1$*, which is appropriate as neither the earth or atmoshere density is changing, and where for* $\beta_{C_m} \approx \beta_{C_0} \approx 1$ *to give*

$$\hat{\beta}_{C_m} \approx \frac{(\rho_m)^{1/3}}{(\rho_m)^{1/3} - (\rho_0)^{1/3}} \approx 1 .$$

as for the earth $\rho_0 << \rho_m$.

Forces in the Chameleon Model

For a small object of mass m placed near the surface of a larger object of mass $M << m$ at a distance r_x, the field-force F_ϕ on the smaller object is given by

$$\vec{F}_\phi \approx mc^2 \left(\beta_m / M_{Pl}\right) \partial \nabla \phi_0 \ \hat{\phi} , \tag{8}$$

where $\hat{\phi}$ denotes the direction of the field force and the change in the gradient $\nabla \phi_0$ of the ambient (atmosphere) Chameleon field is given as

$$\partial \nabla \phi_0 \approx -3 \left(1/4\pi M_{Pl}\right) \left(\Delta R_m / R_m\right) \left(M / r_x^2\right) \left(\hbar / c\right) . \tag{9}$$

where Khoury and Weltman sets the small object's - mass coupling factor $\beta_m \approx 1$. The mass coupling factor β_m infers how well an object couples to the surrounding ambient chameleon field.

THE MODIFIED THINSHELL MODEL

It was noted that equation (7) does not have a means to account for atmospheric variations. Further, it was noted that for the earth

$$\Delta R_{earth} / R_{earth} \approx \sqrt{\frac{l_p}{R_{earth}}} . \tag{10}$$

Since the Chameleon model is a dark energy or quantum scalar model, events must occur on the Planck time. Such that, human units must be artificially imposed when measuring dark energy fluctuations in the thinshell (see for example, Robertson, 2006). Therefore, the average time of a dark energy event in the thinshell of the earth occurs on the Planck time T_{pl} times the square root of the ratio of the observed/measured radius of the earth to the Planck length. That is, in the earth's thinshell, the average dark energy event time

$$t_E \approx T_{pl} \sqrt{\frac{R_{earth}}{l_p}} \approx 3.38x10^{-23} \sec .$$

Whereby, the total energy in the earth's thinshell can be given by

$$E_{\Delta R_{earth}} \approx F_{\phi_{atm}} \Delta R_{earth} \approx \frac{T_{pl}}{t_E} E_p \approx E_p \sqrt{\frac{l_p}{R_{earth}}} \approx 3.12x10^{-12} J \quad (0.0194 \ GeV) ,$$

where is the $F_{\phi_{atm}}$ is the normal ambient field force on the thinshell and E_p is the Planck Energy.

Since the ambient field force $F_{\phi_{atm}}$ on the thinshell will change with changes in the density of the atmosphere surrounding an object, the ratio $\Delta R_m / \bar{R}_m$ is redefined by

$$\frac{\Delta R_m}{\bar{R}_m} \approx \beta_N^2 \sqrt{\frac{l_p}{\bar{R}_m}} , \tag{11}$$

where β_N is defined as the environmental coupling factor, where for the earth $\beta_N^2 \approx 1$.

The event time is then redefined by

$$t_E \approx \frac{T_{pl}}{\beta_N^2}\sqrt{\frac{\bar{R}_m}{l_p}}\,,$$

where the square of the environmental coupling factor

$$\beta_N^2 \approx \frac{F'_{\phi_0}\,\Delta R_m}{E_p}\sqrt{\frac{\bar{R}_m}{l_p}}\,,$$

where F'_{ϕ_0} infers a change from the normal ambient field force of the earth's. That is, the earth environmental coupling factor provides a basis from which environmental changes can be compared and allows such comparisons to all other objects.

Here $\bar{R}_m$ denotes an object's radial factor, which in the general case is the radius of an object. However, in the solid rocket model this will be shown not to be the case.

Rearranging equation (11) gives the thin-shell thickness for any object as

$$\Delta R_m \approx \beta_N^2\sqrt{l_p \bar{R}_m}\,. \tag{12}$$

Now using equations (7) and (12), the environmental coupling factor β_N is given as

$$\beta_N \approx \left(1/3 \left(\frac{M_E^2}{\hat{\beta}_{C_N}\,\rho_N \bar{R}_N \sqrt{l_p \bar{R}_N}} \right) \left(\frac{2M_{PL}^4}{\rho_0} \right)^{1/3} \right)^{1/2}, \tag{13}$$

which allows the coupling factor β_m to be given in similar form as

$$\beta_m \approx \left(1/3 \left(\frac{M_E^2}{\hat{\beta}_{C_m}\,\rho_m \bar{R}_m \sqrt{l_p \bar{R}_m}} \right) \left(\frac{2M_{PL}^4}{\rho_0} \right)^{1/3} \right)^{1/2}, \tag{14}$$

For the earth, equations (13) and (14) are identical. Therefore, letting $\beta_{C_0} \approx 1$, $\hat{\beta}_{C_N} \approx \beta_{C_N} \approx 1$ and $\hat{\beta}_{C_m} \approx \beta_{C_m} \approx 1$ for the earth with a density $\rho_N = \rho_m \approx 5520\ kg/m^3$, $\bar{R}_N = \bar{R}_m = 6.37\times 10^6\ m$ and atmosphere density $\rho_{0_{atm}} \approx 1.2\ kg/m^3$, equation (13) or (14) yield $\beta_N = \beta_m \approx 0.98$, which is ≈ 1 as noted by Khoury and Weltman (2004a and 2004b) and validates equation (12).

Then using equations (13) and (14), the environmental factor $\hat{\beta}_{C_N}$ is given as

$$\hat{\beta}_{C_N} \approx 1/3\,\beta_N^{-2} \left(\frac{M_E^2}{\rho_N R_N \sqrt{l_p \bar{R}_N}} \right) \left(\frac{2M_{PL}^4}{\rho_0} \right)^{1/3}, \tag{15}$$

and the object factor $\hat{\beta}_{C_m}$ to be given in similar form as

$$\hat{\beta}_{C_m} \approx 1/3\,\beta_m^{-2} \left(\frac{M_E^2}{\rho_m \bar{R}_m \sqrt{l_p \bar{R}_m}} \right) \left(\frac{2M_{PL}^4}{\rho_0} \right)^{1/3}, \tag{16}$$

which (using equation (16)) for the earth with $\hat{\beta}_{C_m} = 0.98$ implies that

$$0.98 \approx 1/3 \left(\frac{M_E^2}{\rho_m \bar{R}_m \sqrt{l_p \bar{R}_m}} \right) \left(\frac{2M_{PL}^4}{\rho_0} \right)^{1/3};$$

to give the Planck length

$$l_p \approx \frac{1}{\overline{R}_m^3}\left(\frac{1}{0.98}\left(\frac{M_E^2}{3\rho_m}\right)\left(\frac{2M_{PL}^4}{\rho_0}\right)^{1/3}\right)^2 . \tag{17}$$

For the earth with a density $\rho_m \approx 5520\ kg/m^3$ *,* $\overline{R}_m = 6.37\times 10^6\ m$ *and atmosphere density* $\rho_0 = \rho_{atm} \approx 1.2\ kg/m^3$ *, equations (17) yields* $l_p \approx 1.03x10^{-35} m$ *, which given the estimates on nearly every value in equation (17), is only about a factor of* $\pi/2$ *from the Planck length* $l_p \approx 1.616252x10^{-35} m$ *, further validating equation(12).*

Forces in the Modified Chameleon Model

In the modified Chameleon Model, the densities and corresponding coupling factors are allowed to change – denoted by ∂ , *which does not infer a derivative*. If these changes are one sided, the modified Chameleon model becomes as in Figure 2. Noting that the density change can be in either the smaller or larger object or in the ambient (atmosphere) Chameleon field.

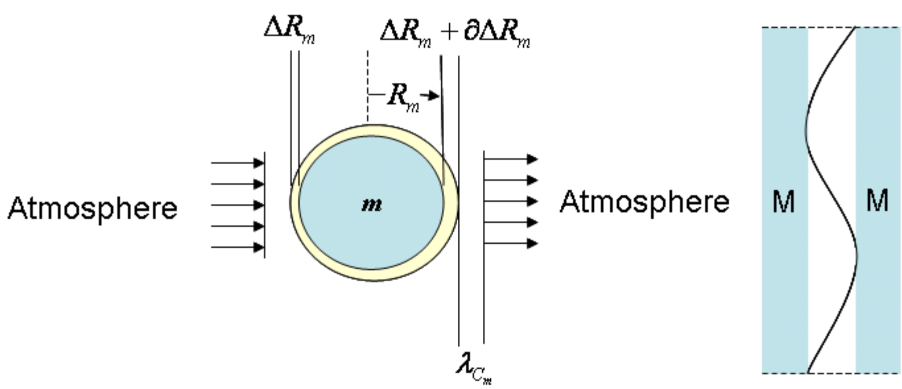

FIGURE 2. Modified Chameleon Model.

The thinshell change $\partial\Delta R_m$ is given from equations (12), as

$$\partial\Delta R_m \approx \partial\beta_N^2 \sqrt{l_p \partial\overline{R}_m} , \tag{18}$$

such that, the coupling factors of equations (13-14) to reflex this change is given by

$$\partial\beta_i \approx \left(1/3\left(\frac{M_E^2}{\partial\hat{\beta}_{C_i}\partial\rho_i\sqrt{l_p\partial\overline{R}_i^3}}\right)\left(\frac{2M_{PL}^4}{\partial\rho_0}\right)^{1/3}\right)^{1/2} \tag{19}$$

and the motion factors of equations (15-16) to reflex this change is given by

$$\partial\hat{\beta}_{C_i} \approx \partial\beta_i^{-2}\left(\frac{M_E^2}{3\partial\rho_i\sqrt{l_p\partial\overline{R}_i^3}}\right)\left(\frac{2M_{PL}^4}{\partial\rho_0}\right)^{1/3} , \tag{20}$$

where $i = N, m$.

The change $\partial\Delta R_m$ in the thinshell infers that the gradient $\nabla\phi_0$ of the ambient (atmosphere) Chameleon field, equation (9), about the smaller object be given as

$$\partial(\nabla\phi_0) \approx -3\left(\frac{1}{4\pi M_{Pl}}\right)\left(\frac{\Delta R_m + \partial\Delta R_m}{\bar{R}_m} - \frac{\Delta R_m}{\bar{R}_m}\right)\left(\frac{M}{r_x^2}\right)\left(\frac{\hbar}{c}\right) = 2M_{Pl}\left(\frac{3\partial\Delta R_m}{\bar{R}_m}\right)\left(\frac{g_N}{c^2}\right), \tag{21}$$

such that, the field force F_ϕ, equation (8), be given as

$$\vec{F}_\phi = mc^2\left(\frac{\partial\beta_m}{M_{Pl}}\right)\partial(\nabla\phi)\ \hat{\phi} \approx 2\partial\beta_m\left(\frac{3\partial\Delta R_m}{\bar{R}_m}\right)mg_N\ \hat{\phi} = 6\partial\beta_m\left(\frac{\partial\Delta R_m}{\bar{R}_m}\right)F_N\ \hat{\phi}, \tag{22}$$

where $g_N \equiv \left(-G_N M/r_x^2\right)$ is the acceleration of gravity due to the large object of mass M producing a Newtonian force $F_N \equiv mg_N$ on the smaller object of mass m.

Using equation (18), the field force of equation (22) is given in terms of the coupling factors as

$$\vec{\boldsymbol{F}}_\phi = 6\partial\beta_m\partial\beta_N^2\sqrt{\frac{l_p}{\bar{R}_m}}\ F_N\ \hat{\phi}\,. \tag{23}$$

The field force of equation (23) can know be easily introduced into the formed $F = (1-\theta)F_N$ used in fifth force searches (see: Robertson, 2009a) by letting the fifth force coefficient

$$\theta \approx \frac{F_\phi}{F_N} = 6\partial\beta_m\left(\frac{\partial\Delta R_m}{\bar{R}_m}\right) = 6\partial\beta_m\partial\beta_N^2\sqrt{\frac{l_p}{\bar{R}_m}}\,. \tag{24}$$

THE SOLID ROCKET MOTOR

The modified Chameleon Model can now be used to develop the thrust from a solid rocket motor. This is illustrated in Figure 3, where the density varies as the propellant mass is exhausted as hot-gas to produce thrust. The dash lines outward from the nozzle indicate a small perturbation in the ambient field density caused by the exhausted mass ∂m_r. This perturbation is neglected in this analysis. Further, the changes to the thin-shells of the two masses are only shown in Figure 3 in the plane of motion as the radial changes counter one another and therefore do not add to the propulsive forces.

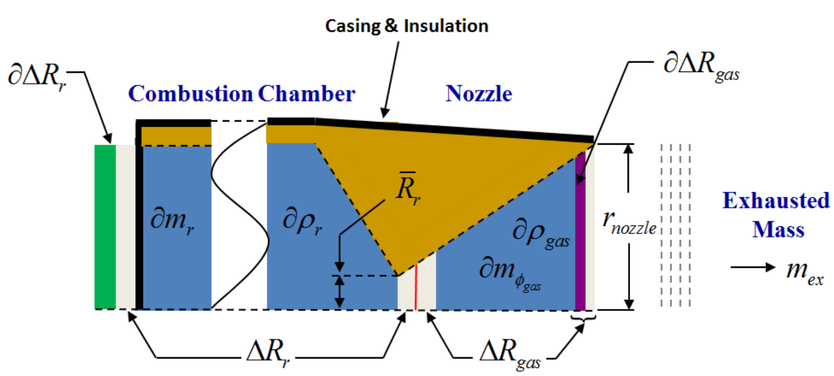

Figure 3. Solid Rocket Motor – Modified Chameleon Model

Field Forces

In this model there are actually two masses, 1) the rocket with changing density $\partial\rho_r$ and radial factor $\bar{R}_r$ and 2) the exhaust hot gas with changing density $\partial\rho_{gas}$ in the nozzle and radial factor $\bar{R}_{gas}$. Using equation (23), the field force associated with the changing rocket density or changing rocket field mass ∂m_{ϕ_r} is given as

$$\vec{\boldsymbol{F}}_r \approx 6\partial\beta_r \left(\partial\beta_{gas} - \partial\beta_{N_r}\right)^2 \sqrt{\frac{l_p}{\bar{R}_r}}\; F_{N_r}\;\hat{\phi}_r \approx \partial m_{\phi_r} a_r\;\hat{\phi}_r = \partial m_{\phi_r} \vec{\boldsymbol{a}}_r \tag{25}$$

and the field force associated with the accelerating gas density or changing gas field mass $\partial m_{\phi_{gas}}$ is given as

$$\overleftarrow{\boldsymbol{F}}_{gas} \approx -6\partial\beta_{gas} \left(\partial\beta_r - \partial\beta_{N_{gas}}\right)^2 \sqrt{\frac{l_p}{\bar{R}_{gas}}}\; F_{N_{gas}}\;\hat{\phi}_r \approx -\partial m_{\phi_{gas}} a_{gas}\;\hat{\phi}_r = \partial m_{\phi_{gas}} \overleftarrow{\boldsymbol{a}}_{gas}\,. \tag{26}$$

where

$$a_{gas} \approx \frac{v_{gas}^2}{2\bar{R}_{gas}}\,. \tag{27}$$

The Newtonian forces on the changing field masses are given as

$$\begin{aligned} \overleftarrow{\boldsymbol{F}}_{N_r} &= -\partial m_{\phi_r} g_N\;\hat{\phi}_r = \partial m_{\phi_r} \overleftarrow{\boldsymbol{g}}_N\,; \\ \overleftarrow{\boldsymbol{F}}_{N_{gas}} &= -\partial m_{\phi_{gas}} g_N\;\hat{\phi}_r = \partial m_{\phi_{gas}} \overleftarrow{\boldsymbol{g}}_N\,. \end{aligned} \tag{28}$$

Coupling Factors

The rocket-hot gas system is viewed as a single body system with phased fifth force coefficients about the rocket due to time dilation and retardation associated with the hot-gas motion (see; Robertson, 2009b). This implies a phase factor φ,

$$\varphi = \frac{\tau}{\tau + \Delta\tau} << 1\,; \tag{29}$$

where the changing mass coupling factors

$$\partial\beta_r \approx \partial\beta_{gas} \approx \frac{1}{6\varphi}\left|\frac{F_r}{F_{N_r}}\right| = \frac{1}{6\varphi}\left(\frac{a_r}{g_N}\right). \tag{30}$$

The hot gas relaxation time

$$\tau \approx \frac{\bar{R}_{gas}}{v_{gas}} \tag{31}$$

and the retardation time

$$\Delta\tau \approx \frac{m_{ex}}{\dot{m}} > \tau\,, \tag{32}$$

where $\dot{m}$ is the propellant mass flow rate through the nozzle and the exhausted mass

$$m_{ex} \approx m_r - \partial m_r\,, \tag{33}$$

where m_r is the initial rocket mass and $\partial m_r \approx m_r - m_{ex}$ is the rocket burnout mass.

Environmental Factors

The environmental coupling factors are given from equation (20) as

$$\partial\beta_{N_r} \approx \left(\frac{1}{3}\left(\frac{M_E^2}{\partial\hat{\beta}_{C_r}\rho_N \bar{R}_N\sqrt{l_p\bar{R}_N}}\right)\left(\frac{2M_{PL}^4}{\rho_0}\right)^{1/3}\right)^{1/2} \tag{34}$$

and

$$\partial\beta_{N_{gas}} \approx \left(\frac{1}{3}\left(\frac{M_E^2}{\partial\hat{\beta}_{C_{gas}}\rho_N \bar{R}_N\sqrt{l_p\bar{R}_N}}\right)\left(\frac{2M_{PL}^4}{\rho_0}^{1/3}\right)\right)^{1/2}; \tag{35}$$

noting that there is no change in the Newtonian mass density ρ_N, radial factor $\bar{R}_N$ and atmosphere density ρ_0 about the SRM. Such that, equations (34) and (35) differ only by the factors $\partial\hat{\beta}_{C_r}$ and $\partial\hat{\beta}_{C_{gas}}$ due to the changing rocket mass densities. These factors are given from equation (20) as

$$\partial\hat{\beta}_{C_r} \approx \frac{1}{3}\partial\beta_r^{-2}\left(\frac{M_E^2}{\partial\rho_r\sqrt{l_p\partial\bar{R}_r^3}}\right)\left(\frac{2M_{PL}^4}{\rho_0}\right)^{1/3}, \tag{36}$$

and

$$\partial\hat{\beta}_{C_{gas}} \approx \frac{1}{3}\partial\beta_{gas}^{-2}\left(\frac{M_E^2}{\partial\rho_{gas}\sqrt{l_p\partial\bar{R}_{gas}^3}}\right)\left(\frac{2M_{PL}^4}{\rho_0}\right)^{1/3}. \tag{37}$$

Changing Densities

The changing rocket field density

$$\partial\rho_r \approx \rho_r' + \left(\frac{F_r}{F_{N_r}}\right)\rho_r' \approx \frac{3\partial m_r}{4\pi\partial\bar{R}_r^3} \;\Rightarrow\; \frac{\partial m_r}{\bar{R}_r^3} + \left(\frac{a_r}{g_N}\right)\left(\frac{m_r}{\bar{R}_r^3}\right) \approx \frac{\partial m_r}{\partial\bar{R}_r^3}, \tag{38}$$

where the normal rocket density change

$$\rho_r' = \frac{3\partial m_r}{4\pi\bar{R}_r^3} \tag{39}$$

and the changing hot gas field density

$$\partial\rho_{gas} \approx \left(\frac{F_{gas}}{F_{N_{gas}}}\right)\rho_{gas}' \approx \frac{3\partial m_{\phi_{gas}}}{4\pi\partial\bar{R}_{gas}^3} \;\Rightarrow\; \left(\frac{a_{gas}}{g_N}\right)\left(\frac{\partial m_{\phi_{gas}}}{\bar{R}_{gas}^3}\right) \approx \frac{\partial m_{\phi_{gas}}}{\partial\bar{R}_{gas}^3}, \tag{40}$$

where the hot gas density change

$$\rho_{gas}' \approx \frac{3\partial m_{\phi_{gas}}}{4\pi\bar{R}_{gas}^3}. \tag{41}$$

Radial Factors

Equation (38) gives the rocket's changing radial factor as

$$\partial\bar{R}_r \approx \left(\frac{F_{N_r}}{F_{N_r} + F_r\left(\frac{m_r}{\partial m_r}\right)}\right)^{1/3}\bar{R}_r = \left(\frac{g_N}{g_N + a_r\left(\frac{m_r}{\partial m_r}\right)}\right)^{1/3}\bar{R}_r \tag{42}$$

and equation (40) gives hot gas radial factor as

$$\partial\overline{R}_{gas} \approx \left(\frac{F_{N_{gas}}}{F_{gas}}\right)^{1/3} \overline{R}_{gas} = \left(\frac{g_N}{a_{gas}}\right)^{1/3} \overline{R}_{gas} . \tag{43}$$

Hot Gas Radial Factor

The hot gas density in the nozzle with respect to a constant (average) gas pressure P_{gas} on the nozzle and constant (average) hot gas velocity v_{gas} is give as

$$\rho'_{gas} \approx \frac{3}{4}\left(\frac{P_{gas}}{v_{gas}^2}\right) \text{ x } 100 , \tag{44}$$

where the factor 100 is a correction factor needed to converts the field mass to human units in order to equate the density of equation (44) back to equation (41).

Now combining equation (44) with equation (41) gives the gas field mass

$$\partial m_{\phi_{gas}} \approx \pi\left(\frac{\overline{R}_{gas}^3}{v_{gas}^2}\right) P_{gas} \text{ x } 100 \tag{45}$$

and gas radial factor

$$\overline{R}_{gas} \approx \left(\frac{1}{100} \cdot \frac{\partial m_{\phi_{gas}} v_{gas}^2}{\pi P_{gas}}\right)^{1/3} . \tag{46}$$

Now letting

$$\frac{1}{100} P_{gas} A_{Nozzle} \approx \partial m_{\phi_{gas}} \left(\frac{v_{gas}^2}{2\overline{R}_{gas}}\right) , \tag{47}$$

where A_{Nozzle} is the nozzle cross sectional area, which when combined with equation (46) yields

$$\overline{R}_{gas}^2 = \frac{2}{\pi} A_{Nozzle} , \tag{48}$$

which simplifies the hot gas radial factor

$$\overline{R}_{gas} = \left(\frac{2 A_{Nozzle}}{\pi}\right)^{1/2} = r_{nozzle}\sqrt{2} . \tag{49}$$

As will be shown in the example, the rocket radial factor $\overline{R}_r$ is approximately the radius of the throat, which is appropriate as the throat is the only area about the surface of the rocket (excluding the nozzle) that is changing due to particulate motion in the direction of motion.

Thrust

The total force or thrust of the SRM is given from the fifth force model (see: Robertson, 2009a) at propellant burnout as

$$\text{Thrust} \approx \sum \theta \cdot F_{N_r} = \sum \theta \cdot \left(\partial m_r g_N\right) , \tag{50}$$

where the sum of the fifth force coefficients $\sum \theta_r$ are given with respect to equations (24-26) as

$$\sum \theta_r = \theta_r + \theta_{gas} = \frac{F_r}{F_{N_r}} + \frac{F_{gas}}{F_{N_{gas}}}$$
$$= \left(6\partial\beta_r \left(\partial\beta_{gas} - \partial\beta_{N_r}\right)^2 \sqrt{\frac{l_p}{\bar{R}_r}} - 6\partial\beta_{gas} \left(\partial\beta_r - \partial\beta_{N_{gas}}\right)^2 \sqrt{\frac{l_p}{\bar{R}_{gas}}} \right). \quad (51)$$

EXAMPLE

This example uses the data from example 2-1 in Sutton and Ross (1976) converted to metric units. The example appears to be a Sidewinder, AMRAM or Similar Missile. Figure 4 is an AMRAM missile given to show the general dimensions. Table I are the given parameters, the parameters surmised from a Sidewinder missile, and the parameters calculated from these values. The parameters of Table I was used to calculate the parameters of the rocket mass in Table II and the parameter of the hot gas in the nozzle in Table III, and the parameters of the fifth force in Table IV. One assumption made is that the total missile weight is equivalent to the solid rocket motor weight. The burn out weight given was $\bar{F}_N \approx 578.74\ N$ and the thrust was given as $\bar{F}_r \approx 2.49 \times 10^4\ N$.

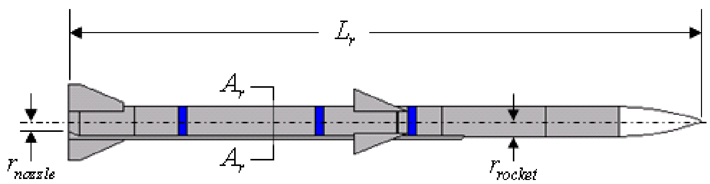

Figure 3. General Missile Dimension

Table I. The given, surmised and calculated values for the hot-gas rocket.

Given	Surmised	Calculated
$m_r = 90.72\ kg$	$r_{rocket} \approx 0.063\ m$	$A_r \approx 0.0126\ m^2$
$m_{ex} = 31.75\ kg$	$L_r \approx 2.98\ m$	$V_r \approx 0.0377\ m^3$
$v_{gas} \approx 2355.49\ m/s$	$r_{nozzle} \approx 0.051\ m$	$\rho_r \approx 2403.83\ kg/m^3$
$\dot{m} = 10.58\ kg/s$	$P_{gas} \approx 1.97 \times 10^6\ N/m^2$	$A_{nozzle} \approx 0.0081\ m^2$

Table II. Parameters of the Rocket Mass

Parameter	Equation	Parameter	Equation
$\bar{R}_r \approx 0.012928\ m$	42	$\Delta\tau \approx 3\ s$	32
$\partial\bar{R}_r \approx 0.0032\ m$	42	$\partial\beta_r \approx 7.06 \times 10^5$	30
$\partial m_r \approx 58.97\ kg$	33	$\partial\hat{\beta}_{C_r} \approx 0.0018$	36
$\partial\rho_r \approx 1.89 \times 10^8\ kg/m^3$	38	$\partial\beta_{N_r} \approx 21.02$	34

Table III. Parameters of the Hot Gas in the Nozzle

Parameter	Equation	Parameter	Equation
$\partial m_{\phi_{gas}} \approx 0.041\ kg$	45	$\partial\rho_{gas} \approx 4.39 \times 10^8\ kg/m^3$	40
$\bar{R}_{gas} \approx 0.0718\ m$	46/49	$\partial\beta_{gas} \approx 7.06 \times 10^5$	30
$a_{gas} \approx 3.86 \times 10^7\ m/s^2$	27	$\partial\hat{\beta}_{C_{gas}} \approx 0.033$	37
$\rho'_{gas} \approx 26.63\ kg/m^3$	31/44	$\partial\beta_{N_{gas}} \approx 4.89$	35

$\partial\overline{R}_{gas} \approx 4.55\times10^{-4}\ m$	43	$\tau \approx 3.05\times10^{-5}\ s$	31

Table IV. Parameters of the Fifth Force on the rocket

Parameter	Equation
$\varphi \approx 1.02\times10^{-5}$	26
$F_{N_{gas}} \approx 1.59x10^{6}\ N$	28
$\theta_r \approx 74.79$	51
$\theta_{gas} \approx -31.74$	51
$\sum\theta \approx 43.05$	51
Thrust ≈ 24900 N *	50

*The radial factor $\overline{R}_r$ was adjusted to give the correct thrust, but is on the order of the expected nozzle throat.

CONCLUSION

The Khoury and Weltman (2004a; 2004b) Chameleon Model presents a small force addition to gravity on earth, but has large consequences on the galactic scale, which infers a strong dark matter/energy connection. This paper shows that a slightly Modified Chameleon model can be used to predict the thrust of a solid rocket motor. As such, extension of this model to a realistic engineering - space vacuum propulsion system should be achievable - one using varying mass densities versa mass expulsion.

The author notes that this model needs further development. Specially, further verification in real rocket analysis is warranted.

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